Heuristic optimization of short corbels by smeared cracking finite element analysis

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Abstract

This paper deals with the economic optimization of reinforced concrete short corbels typically used in construction. The study shows the efficiency of heuristic optimization by the random search, the descent local search and the Simulated Annealing (SA) algorithms. The evaluation of solutions follows a nonlinear finite element analysis with smeared cracking up to failure. Designs are considered feasible when they withstand a prescribed reference load. The algorithms are applied to a typical short corbel of 0.350m of total depth for which there is available data about the mode of failure and ultimate loads. The distance of the applied load to the built-in section is 0.150m. This example has seven discrete design variables for the geometry of the corbel, material and passive reinforcement. The application of the SA algorithm requires the calibration of the initial temperature and threshold, the number of variables modified in each iteration, the length of the Markov chains and the reducing coefficient. Each heuristic is run nine times so as to obtain statistical information about the minimum, average and deviation of the results. The best result has a cost of 10.4770€ for the SA algorithm. Finally, solutions and run times indicate that heuristic optimization is a forthcoming option for the design of real non-linear finite element analysed structures.
Keywords: economic optimization, heuristics, concrete structures, structural design, smeared cracking.

WIT Transactions on The Built Environment, Vol 125,
www.witpress.com, ISSN 1743-3509 (on-line)
doi:10.2495/OP120071

1 Introduction

The emergence of heuristic optimization techniques was a consequence of artificial intelligence procedures. These approximate techniques are appropriate for optimizing realistic structures because they often find a fast and near global optimal solution. Heuristic approaches include several algorithms such as genetic algorithms, simulated annealing, particle swarm optimization, and ant colony optimization, inter alia. A recent review of heuristics for project and construction management can be found in the study by Liao *et al*. [1]. Cohn and Dinovitzer [2] provided an extensive state-of-the-practice in structural optimization noting that most of the research studies focused mainly on steel structures, whereas only few dealt with concrete structures. The history of heuristic optimization in RC structures can be traced back to the late 1990s. From them on, many studies based on evolutionary computation have been applied for optimizing structural concrete problems, especially genetic algorithms. The present authors' research group has recently reported on non-evolutionary algorithms to the fully automated structural design and cost optimization of realistic three-dimensional structures such as walls [3–9].

Building on this research line, this paper proposes an automatic design method for reinforced concrete short corbels using heuristic optimization techniques as well as a smeared cracking model for concrete fracture. This method requires two basic tools: (i) a numerical analysis module capable of evaluating the ultimate failure load of the structure in different load states considering progress smeared cracking criteria, and (ii) an optimization module capable of performing an iterative variation of the current design and evaluating the objective function to accept or reject the modified design.

A realistic short corbel is analyzed to validate the numerical analysis module. The realistic structures have been tested in the laboratory so the real deformation and ultimate failure loads results results are available. The calculation module is based on a finite element program called FINEL, where several sub-modules to analyze the concrete smeared cracking have been coded. On the other hand, the optimization module is based on four heuristic algorithms.

2 Numerical analysis module

The numerical analysis module is validated using the experimental ultimate failure load of eight short corbels performed by Montenegro [10]. This work analyzed the failure mechanisms depending on the amount of reinforcing steel as well as the onset of cracking load level. In addition, relations between applied load and reinforcement deformation for the corbels tested are established. Figs. 1 and 2 show the dimensions and the reinforcement setup of the corbels tested. The concrete mix design, the curing process and the instrumentation used during the test can be found in the mentioned reference. Note that the loading process has been gradual, starting with 20kN loading steps until the appearance of the first cracks and then stepped up to 50kN load, until the corbel ruin.

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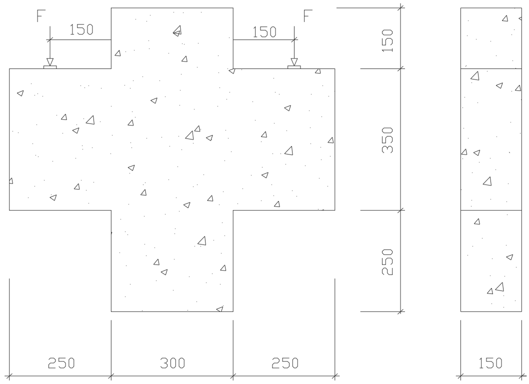

Figure 1: Short corbel geometry.

Table 1 includes the reinforcement setup of the corbels tested. The nomenclature of the corbels has been adopted in the form "CHϕVϕ", where C stands for "*Consolo*" (corbel in Portuguese), Hϕ and Vϕ represents the vertical and horizontal stirrups, respectively. Table 2 summarizes the onset of cracking load results and the ultimate failure load for the different tests. As noted above, the software used to check the ultimate failure load was FINEL which modular structure has allowed to use some of its calculation modules and adapt them to the smeared cracking scheme, particularly altering aspects of the material non-linearity. The original FINEL modules used were the mesh generation, the nodal renumbering, the boundary conditions and the Choleski method to solve equations. Issues related to non-linearity was reflected in the rigidity matrix, non-linear relationships, residual stresses, cracking, etc. Two auxiliary modules, which were made by the third author [11], were also used to process and obtain graphs representing a section of mesh reference plane parallel to the XZ and contains Gauss point closest to the plane XZ.

Table 1: Number and diameter of the reinforcement setup.

Corbel	**Reinforcement setup (mm)**					
	N1	**N2**	**N3**	**N4**	**N5**	**N6**
CH0V0	6 ϕ 12.5	11 ϕ 5	4 ϕ 12.5	4 ϕ 12.5	-	-
CH5V5	"	"	"	"	4 ϕ 5.0	8 ϕ 5.0
CH5V0	"	"	"	"	4 ϕ 5.0	-
CH0V5	"	"	"	"	-	8 ϕ 5.0
CH4V0	"	"	"	"	4 ϕ 4.2	-
CH4V4	"	"	"	"	4 ϕ 4.2	8 ϕ 4.2
CH6V0	"	"	"	"	4 ϕ 6.3	-
CH4V4*	"	"	"	"	4 ϕ 4.2	4 ϕ 4.2

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Table 2: FINEL model prediction versus laboratory test results.

TEST	Laboratory test results [10]			FINEL model prediction (kN)			
	Cracking load (kN)		Ultimate failure load (kN)				
	Onset of cracking	Onset of strut		First crack rising	Strut cracking rising	Ultimate failure load (kN)	F_r/F_c
CH0V0	200	400	1000	240	480	1080	0.93
CH5V5	160	400	1250	180	448	1080	1.16
CH5V0	160	400	1070	180	480	1050	1.02
CH0V5	200	400	965	180	450	1065	0.91
CH4V4	180	320	1080	180	420	1080	1.00
CH4V0	180	400	1160	180	480	1035	1.12
CH6V0	200	350	1195	240	540	1150	1.04
CH4V4*	160	250	790	120	360	915	0.86

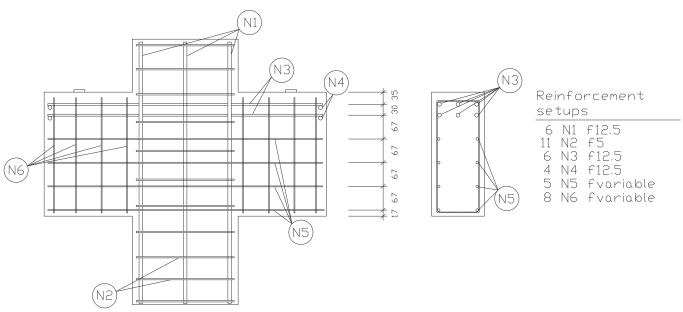

Figure 2: Reinforcement setup.

Figure 3 shows the finite element modeling of a corbel of 250mm long, 150mm wide and 350mm height tested in the laboratory. The total mesh consists of 25 elements for concrete and 79 elements for steel. The reinforcing steel is arranged in the edges of the concrete elements that surrounds them. Half of the corbel has been analyzed given the existence of a symmetry plane of both geometry and loading. Steps were applied using a 15kN load uniformly distributed applied in the 23 and 24 finite elements of the top face.

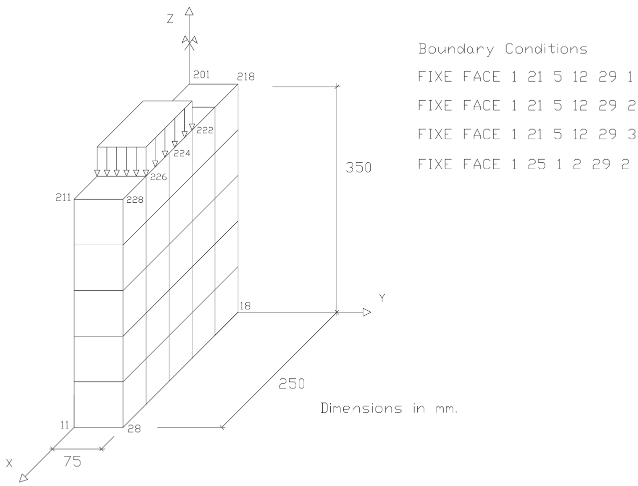

Figure 3: Finite element modelling and boundary conditions.

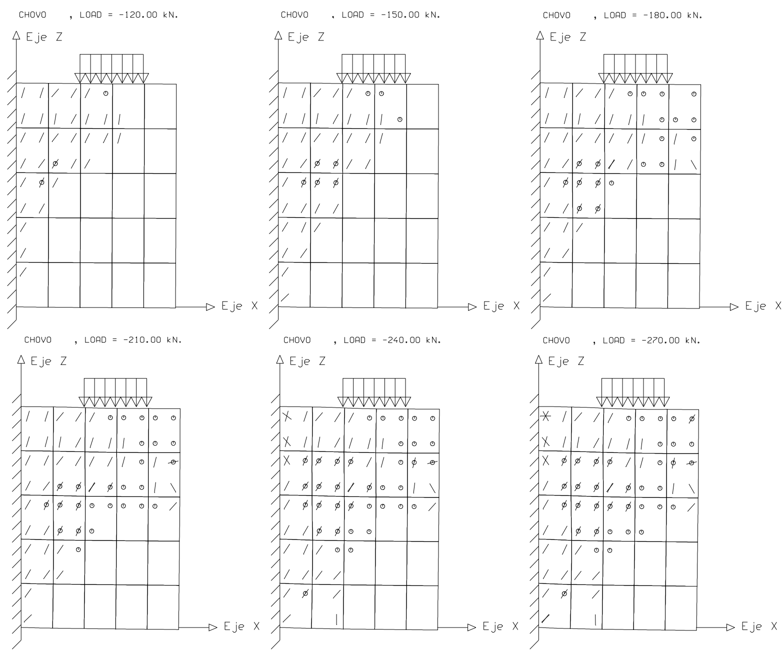

Figure 4: Graphical representation of the cracking development using the numerical analysis modulus. Case: CH0V0, ultimate failure load: 2 x 270kN.

Table 2 summarizes the results obtained using FINEL for predicting the ultimate failure loads as well as the values for which the first crack and the strut cracking appear. The calculated ultimate failure load according to the last converged load step has been included as well. The results obtained with FINEL are of high quality both in regard to obtaining the ultimate failure load as determining the cracking development.

shows a graphic example of the output results of the numerical analysis modulus for a specific case.

One of the main disadvantages of FINEL is the computation time required for each iteration, which normally has hovered around three minutes. This drawback has been forced to limit the maximum number of iterations to be run rather than imposing improving conditions on the objective function for a specified number of iterations.

3 Optimization problem definition

The structural optimization problem deals with the minimization of the objective function F of expression (1), satisfying also the restrictions of expressions (2).

$$F(x_1, x_2, \ldots x_n) = \sum_{i=1,r} p_i * m_i(x_1, x_2, \ldots, x_n) \tag{1}$$

$$g_j(x_1, x_2, \ldots\ldots x_n) \leq 0 \tag{2}$$

$$x_i \in (d_{i1}, d_{i2}, \ldots., d_{iq}) \tag{3}$$

Note that F in expression (1) is the sum of unit prizes multiplied by the measurements of construction units (concrete, steel, formwork, etc.), and that the restrictions on expression (2) are all the structural constraints. The design variables, x_1, x_2,..., x_n, take the discrete values in a list in expression (3). The analysis includes seven discrete variables: the corbel width (B), the characteristic strength of concrete (f_{ck}), and the longitudinal and transverse reinforcement (A_{s1}, A_{s2}, A_{s3}, A_{s4} and A_{s5}) as shown in Fig. 5.

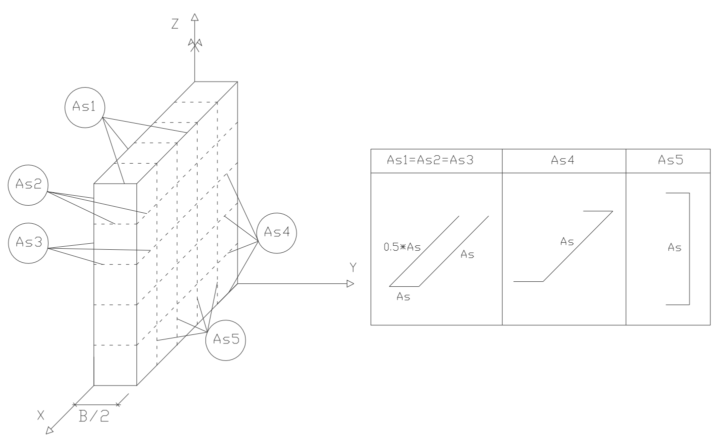

Figure 5: Corbels reinforcement setup.

The parameters of the analysis are all fixed quantities and therefore they are not subject to optimization. Tables 3 and 4 provide details of those parameters for the corbels analyzed. The ultimate failure load is the only restriction imposed on the structure which has to be equal or greater than 500kN. Note that for calculating the failure load provided by FINEL no material safety factor has been considered. Thus, the failure load is the actual resistance load of the corbel.

Table 3: Main parameters of the analysis.

Parameter	Notation	Description	Value
Geometric parameters	L	Corbel length	250 mm
	C	Corbel depth	350 mm
	a	Load application distance	150 mm
Mechanical parameters	f_{yk}	Modulus of elasticity of steel	479 MPa
Loading parameters	P	Loading step	2.0 N/mm^2

Table 4: Basic prizes of the cost function.

Unit	Cost (€)
kg of steel (B-500S)	1.25
m^2 of shuttering and stripping formwork	22.75
m^3 of concrete HA-25	77.80
m^3 of concrete HA-30	82.34
m^3 of concrete HA-35	98.03
m^3 of concrete HA-40	105.17
m^3 of concrete HA-45	111.72
m^3 of concrete HA-50	118.26

4 Optimization module

The optimization module used in this study is based on four heuristic search algorithms: the random search (RS), the directional movement greedy best-first search (DGLS), the descent local search (DLS) and the simulated annealing (SA) method. The algorithms were coded in Visual Basic 6.0 and computer runs were performed in a conventional PC computer with an Intel Pentium of 2.80GHz.

RS consists of generation solutions by random choice of the optimization problem variables. The cost function is evaluated for each solution and then it is checked if the solution satisfies the structural constraints. The computation time has limited the number of iterations performed, which has been 200. Among the feasible solutions found, the lowest cost solution is chosen. Figure 6 shows the results for 200 iterations of RS. The best result found has a cost of 13.5737€ with a computation time of 10 hours.

DGLS involves the progressive reduction of the cost of the objective function from an initial maximum cost solution. The initial solution is modified

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iteratively. If the change produces a better solution and fulfills the condition of minimum ultimate failure load, then the new solution replaces the previous one. The local changes of the current solution are performed by varying one (D1), three (D3) or five variables (D5) simultaneously in order to determine the most efficient movement. The mechanism that modifies a solution defines the neighborhood of this solution. The movements of each variable always move in the direction of decreasing costs so that no variables are allowed to increase its value at any time. 120 iterations have been executed, storing all lower cost solutions found by this algorithm. Figure 7 shows the comparative results of cost trends for D1, D3 and D5.

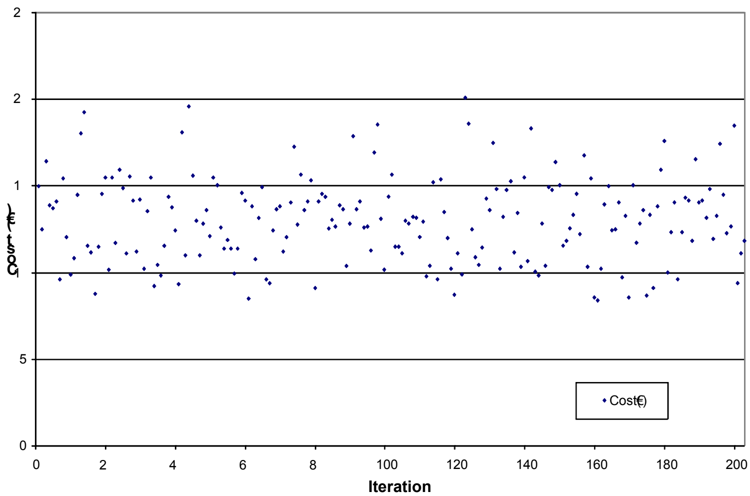

Figure 6: Cost results for a standard RS for 200 iterations.

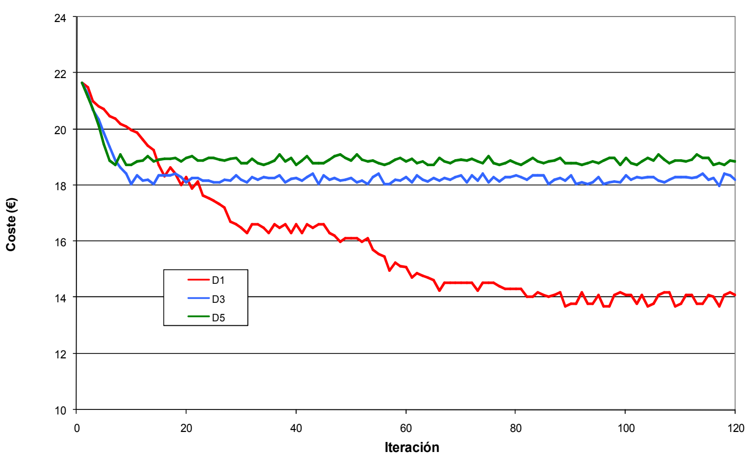

Figure 7: Cost variation for DGLS.

DLS is a greedy best-first search, but unlike the previous method, DLS starts with a random feasible solution and the movements of each variable are allowed to move in any the direction. Unlike DGLS, DLS is a non-directional movement greedy best-first search is an iterative algorithm that starts with a random feasible solution and then attempts to find a better solution by changing some variables of the current solution. If the change produces a better solution, this new solution replaces the previous one, repeating the movements until no further improvements can be found. DLS involve modifying a starting feasible solution iteratively through an appropriate mechanism. From this solution, a random change is applied to the values of the variables. The local changes of the current solution are performed by varying one (G1), three (G3) or five variables (G5) simultaneously, but now allowing random non-directional changes in each chosen variable. The new solution is evaluated and, if this solution reduces the cost and is feasible then this solution replaces the previous one. The starting solution selected to begin with DLS was the best solution found in the RS. A usual stopping criterion used in these iterative local search methods is to finish the movements until a maximum number of iterations without improvement. Here, the number of iterations without improvement is limited to 120 due to the high computation time required. Figure 8 shows the evolution of the cost for each of the movements G1, G3 and G5 applied to DLS. The best solution found with DLS has a cost of 11.9775€ with 7 hours of computation time, being D5 the most effective movement.

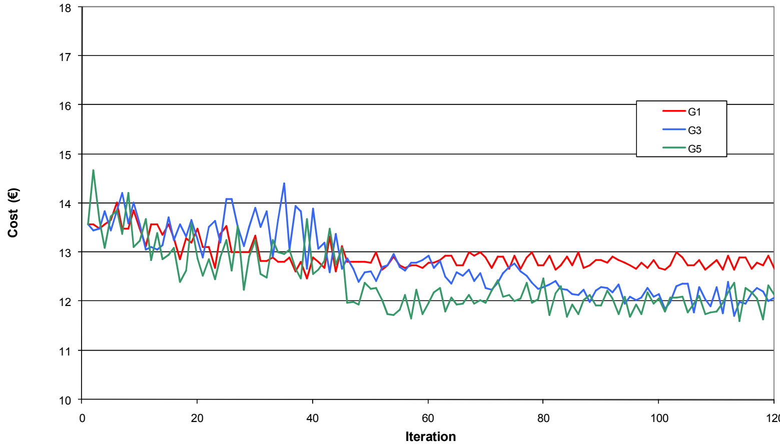

Figure 8: Cost variation for DLS.

The fourth algorithm used in this work was the well-known SA algorithm, that was originally proposed by Kirkpatrick *et al.* [12]. SA is used in global optimization problems to find a good approximation solution in a large search space. SA is based on the analogy of crystal formation from masses melted at high temperature and let cool slowly. The process is governed by Boltzmann expression *exp(-ΔE/T)*, where *ΔE* is the increment of energy of the new configuration and *T* is the temperature. The algorithm starts with a feasible solution randomly generated and a high initial temperature. The initial working solution is changed by a small random move of the values of the variables. The new current solution is evaluated in terms of cost. Greater cost solutions are accepted when a 0 to 1 random number is smaller than the expression *exp(-ΔE/T)*, where *ΔE* is the cost increment and *T* is the current temperature. The current solution is then checked against structural restrictions and if it is feasible, it is adopted as the new working solution. The initial temperature is decreased geometrically (*T=kT*) by means of a coefficient of cooling *k*. A number of iterations called Markov chains is allowed at each step of temperature. The algorithm stops when the temperature is a small percentage of the initial temperature (typically 1%). The SA method is capable of surpassing local optima at high-medium temperatures and gradually converges as the temperature reduces to zero. The SA method requires calibration of the initial temperature, the length of the Markov chains and the cooling coefficient. The initial temperature was adjusted following the method proposed by Medina [13], which consists in choosing an initial value and checking whether the percentage of acceptances of higher energy solutions is between 20-40 percent. If the

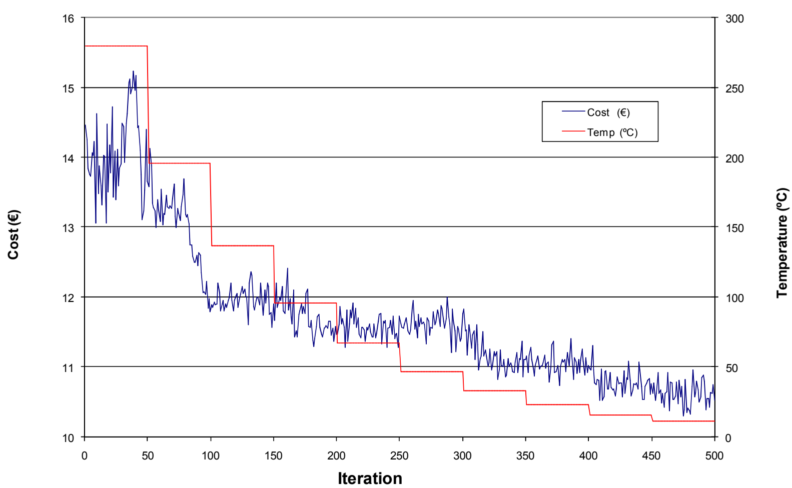

Figure 9: Typical cost variation and temperature reduction for SA.

percentage is greater than 40%, the initial temperature is halved; and if it is smaller than 20%, the initial temperature is doubled. The movement selected for SA was G3. The initial temperature calibrated was 280, the Markov chain length was 50 and the cooling rate was 0.7. These calibration data provide 500 iterations with a computer time of 20 hours. Computer runs were performed 9 times so as to obtain minimum, mean and standard deviation of the random results. The best solution found with SA has a cost of 10,4770€ with 25 hours of computation time Figure 4.6 shows a typical cost evolution for the SA.

5 Results from numerical experiments

Table 5 summarizes the results of best cost and computer running times of the four heuristics used in the research. It can be seen that the lowest cost solution found by SA improves by 22.81%, 27.28% and 12.53% the best results of RS, DGLS and DLS, respectively.

Table 5: Best solutions found by the heuristics.

Algorithm	**Cost (€)**	**Computation time (hours)**
RS	13.5737	10.00
DGLS	14.4072	8.25
DLS	11.9775	7.40
SA	10.4770	25.60

The optimized corbel uses concrete with the highest permitted characteristic resistance (50MPa). Its width is very strict, 180mm and the reinforcing steel setup at the top of the corbel works as a tie. The optimized-cost corbel does not require longitudinal (A_{s5}) nor transverse reinforcement (A_{s4}). The ultimate failure load is 504kN.

6 Conclusions

In this paper we describe an automatic method for the design of real nonlinear finite element analysed reinforced concrete short corbels using heuristic optimization techniques as well as a smeared cracking model for concrete fracture. The results obtained are very close to those obtained experimentally both in regard to obtaining the ultimate failure load as determining the cracking development The analysis reveals that SA is an efficient algorithm for the optimal design of real corbels. In addition, a computer time reduction for the calculation module is required in order to apply other stopping criteria for the heuristics rather than a predetermined number of iterations. The optimized-cost corbel has a very strict width, use concrete with the highest permitted characteristic resistance (50MPa) and does not require longitudinal nor transversal reinforcing steel. Nevertheless, future studies should include additional structures, such as bridge braces and pile plates; and a restatement of the calculation modules to apply the methodology to other more complex structures such as prestressed concrete.

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Acknowledgements

This study was funded by the Spanish Ministry of Science and Innovation (Research Project BIA2011-23602) and the European Community with the FEDER funds.

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