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# Kriging-based heuristic optimization of a continuous concrete box-girder pedestrian bridge

V. Penadés-Plà
*Institute of Concrete Science and Technology (ICITECH), Universitat Politècnica de València, Valencia, Spain*

T. García-Segura
*Department of Construction Engineering and Civil Engineering Projects, Universitat Politècnica de València, Valencia, Spain*

V. Yepes & J.V. Martí
*Institute of Concrete Science and Technology (ICITECH), Universitat Politècnica de València, Valencia, Spain*

ABSTRACT The structural optimization aims to determine the best solutions for the project objectives while guaranteeing the structural constraints. The heuristic algorithms follow an intelligent process in which the design variables are modified for the purpose of optimizing the objective function and verify the constraints. Methodologies like metamodel-based design optimization or surrogate-based optimization carry out a pseudo-optimization applicable to structures. The kriging method provides a response surface from the sample that can be optimized. In this paper, conventional heuristic optimization and kriging-based heuristic optimization are applied to the same case study. This case involves a continuous box-girder pedestrian bridge. The comparison of the methodologies shows the advantages and disadvantages of both methodologies. Furthermore, a major compression of both processes gain a better understanding of the methods and the most suitable cases.

## 1 INTRODUCTION

Bridges are one of the most significant structures in civil engineering. Box-girder bridges can be constructed using many methods as cast in situ or precast in segments and then erected and prestressed (Sennah & Kennedy 2002). Besides, this type of bridge has been found in beam, portal frame, arc, cable-stayed, and suspensions bridges. Its strength against positive and negative bending moments and also to torsional stresses combined with a low dead load have led this type of bridge to be one of the most widespread used nowadays. Therefore, much research has been done to promote understanding and achieve a proper design.

The main objective of structural engineering is to achieve the maximum safety with the minimum investment. This goal is not easy as long as the structural problem is characterized by a wide variety of variables with multiple combinations. In conventional design, the bridge designer decides the over-all structural design according to the topographical and traffic conditions, and the cross section geometry is defined based on design criteria and professional experience. This decision conditions the other variables that are adjusted to guarantee structural safety. The post-tensioned steel and reinforced steel are designed according to the restriction established by the codes. Therefore, only the designers with a large experience obtain an economical, safe, and simple design.

Heuristic optimization process is presented as an alternative to achieve a solution inside the design space that reaches the objective according to the constraints imposed by the codes. This technique has been used to optimize many types of structures, such as precast concrete floors (de Albuquerque et al. 2012), steel reinforced concrete columns (Park et al. 2013), reinforced concrete columns (Nigdeli et al. 2015; Park et al. 2013), reinforced concrete frames (Camp & Huq 2013), reinforced concrete I-beams (García-Segura et al. 2014), prestressed concrete precast road bridges (Martí et al. 2016), and post-tensioned concrete box-girder bridges (García-Segura et al. 2015, 2017). Due to the high complexity of the structural optimization problems, heuristic or metaheuristic algorithms have the best behavior, obtaining a solution with a lower computational cost. However, the structural optimization problems depend on a large number of design variable with various constrains. This causes that the computational cost remains excessive (Simpson et al. 2004). One effective solution to reduce the computational cost of the optimization is the use of approximate response surfaces obtained by surrogate models or metamodels. One of the most encouraging metamodels used in the structural optimization is the kriging model (Cressie 1990). This model provides an optimal interpolation based on regression against observed values of surrounding data points, weighted according to spatial covariance values.

In this study, a three-span continuous box-girder pedestrian bridge will be cost-optimized in two different ways: first using a conventional heuristic optimization, and then using a metamodel-based heuristic optimization based on kriging. Finally, a comparison between both methodologies will be shown and the convenience of kriging model for the robust design will be discussed.

## 2 OPTIMIZATION

Optimization is a process that tries to find the best possible solution. This problem is defined by one or several objective functions, $f$ (mono-objective or multi-objective) that satisfy some constraints, $g_j$.

$$f(x_1, x_2, x_3, \ldots, x_n) \quad (1)$$

$$g_j(x_1, x_2, x_3, \ldots, x_n) \leq 0 \quad (2)$$

where $x_1, x_2, x_3, \ldots, x_n$ are the design variables chosen for the formulation.

Optimization problem is carried out by means of algorithms that establish a set of rules to be followed in solving operational problems. Optimization algorithms can be divided in exact algorithms and heuristic algorithms. On the one hand, exact algorithms reach the global optimum. On the other hand, heuristic algorithms achieve good solutions without guaranteeing the global optimum, but with a lower computational cost. Complex optimization problems like structural optimization are defined for a large number of design variables, and thus, the heuristic algorithms have the best behavior to solve this kind of problems.

Heuristic algorithms try to simulate simple events observed in the nature. In general, the traditional heuristic algorithms look for a local optimum, while the metaheuristic algorithms have tools to avoid local optimums to find a better solution. Metaheuristic algorithms follow an iterative process in which a solution of the problem is defined to after evaluate aptitude by objective function (Figure 1). Last years, some metaheurisitc algorithms have been applied to structural optimization such as variable neighborhood search (Molina-Moreno et al. 2017), ant colony optimization (Martinez-Martin et al. 2013), threshold function (Kutylowski & Rasiak 2014), memetic algorithm (Martí et al. 2015), glowworm swarm algorithm (García-Segura et al. 2014; Yepes et al. 2015) and simulated annealing (García-Segura & Yepes 2016) among others.

However, despite the advances in technology, the computational cost of the structural optimization is still very high. This high computational cost can be reduced by means of the so called surrogate models

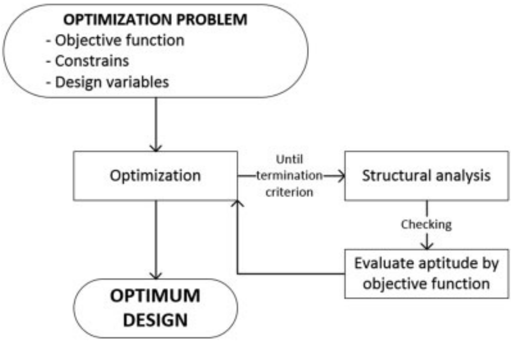


Figure 1. General flow chart of conventional heuristic optimization process.

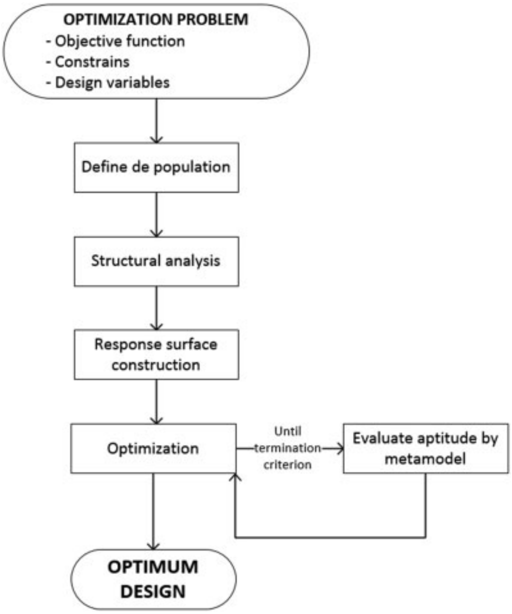


Figure 2. General flow chart of metamodel-based heuristic optimization.

or metamodels (Simpson et al. 2004). These models obtain a response surface from a set of points of the design space to predict more quickly the objective function of the optimization problem. The most used models are: polynomial-based response surface model, the neural networks based surrogate model, and the kriging model. The polynominal-based response surface model is likely awkward in complex engineering problems, and the neural network-based model requires many sample points and much computational time for the training (Forrester & Keane 2009). The kriging model is one promising metamodel as it is more flexible than polynominal-based models and it is not as complicated and time consuming as neural network-based techniques (Li et al. 2010). Figure 2 shows the general flow chart of the metamodel-based heuristic optimization.

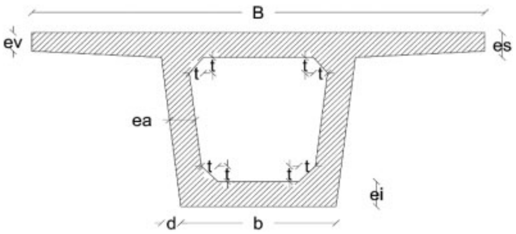


Figure 3. Box-grider cross section.

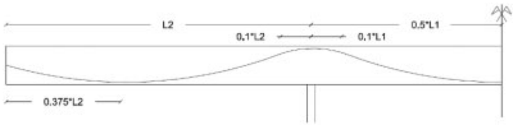


Figure 4. Pedestrian bridge and duct layout.

# 3 CASE OF STUDY

## 3.1 *Pedestrian bridge description*

The structure of the study is a concrete box-girder pedestrian bridge deck with three spans of 40-50-40 meters length (following the relation in which the external span is 80% of the central span). The width of the pedestrian bridge is 3.5 meters. The other geometrical conditions of the cross section are defined by seven variables (Figure 3): the depth ($h$), the width of the bottom slab ($b$), the width of the web inclination ($d$), the thickness of the top slab ($es$), the thickness of the external cantilever section ($ev$), the thickness of the bottom slab ($ei$), and the thickness of the webs ($ea$). This study proposes 15 cm as the minimum thickness. The haunch ($t$), is calculated from the values of other variables (equation 3) according to Schlaich and Scheff's (1982) recommendation. In addition, the haunch must provide the space to contain the ducts in the high and low points.

$$t = max\left\{\frac{b-2\cdot ea}{5}, ei\right\} \tag{3}$$

The strength of the concrete is defined by de variable $f_{ck}$, that can take a value inside a range between 35 MPa and 100 MPa.

The post-tensioned steel formed by 0.6 inch strands is prestressed to 195.52 kN. The ducts are symmetrically distributed through the webs by a parabolic layout. The maximum eccentricity is presented where the bending moment is the maximum or the minimum (Figure 4). At these points, the distance between the duct and the reinforcing bars is 1.5 times the duct diameter. The distance from the piers to the point of inflection is defined by the 10% of the length of each span.

Traditional scaffolding is used in the construction. Table 1 defines other conditions followed in this study such as the materials, the loads actions on the structure, the exposure class and the codes used.

Table 1. Main parameters of the analysis.

| | |
|---|---|
| **Material parameters** | |
| Maximum aggregate size | 20 mm |
| Reinforcing steel | B-500-S |
| Post-tensioned steel | Y1860-S7 |
| Strand diameter | $\Phi_s = 0.6$" |
| Tensioning time | 7 days |
| **Geometrical parameters** | |
| Pedestrian bridge width | B = 3.5 m |
| Number of spans | 3 |
| Central span length | L1 = 50 m |
| External span length | L2 = 40 m |
| Clearance | 5 m |
| Diaphragm thickness | 1.2 m |
| **Exposure related parameters** | |
| External ambient conditions | IIb |
| **Code related parameters** | |
| Code regulation | EHE-08/IAP-11 |
| Service working life | 100 years |
| **Loading related parameters** | |
| Reinforced concrete self-weight | 25 kN/m$^3$ |
| Asphalt layer self-weight | 24 kN/m$^3$ |
| Mean asphalt thickness | 47.5 mm |
| Bridge railing self-weight | 1 kN/m |
| Live load | 4 kN/m$^2$ |
| Differential settling | 5 mm |

## 3.2 *Problem description*

In this study, the problem of the concrete box-girder pedestrian bridge deck optimization involves a single-objective optimization of the cost of the structure. Hence, this optimization aims to minimize the cost (equation 4) and satisfy the constraints (equation 5).

$$Cost = f(x_1, x_2, x_3, \dots, x_n) \tag{4}$$

$$g_j(x_1, x_2, x_3, \dots, x_n) \leq 0 \tag{5}$$

where $x_1, x_2, x_3, \dots, x_n$ are the design variables.

The objective function evaluates the cost for the total number of construction units considering material and placement cost listed in equation 6. Unit prices ($p_i$), shown in Table 2 were obtained from the BEDEC ITEC database (Catalonia Institute of Construction Technology 2016). This database was created by the Institute of Construction Technology of Catalonia (Spain). The data is related to building, urbanism and civil engineering. It contains 750,000 items and provides the commercial costs for 83 Spanish companies.

Concrete unit prices were determined for each compressive strength grade according to the mix design,

Table 2. Unit prices.

| Unit measurements | Cost (€) |
|---|---|
| $m^3$ of scaffolding | 10.02 |
| $m^2$ of formwork | 33.81 |
| $m^3$ of lighting | 104.57 |
| kg of steel (B-500-S) | 1.16 |
| kg of post-tensioned steel (Y1860-S7) | 3.40 |
| $m^3$ of concrete HP-35 | 104.57 |
| $m^3$ of concrete HP-40 | 109.33 |
| $m^3$ of concrete HP-45 | 114.10 |
| $m^3$ of concrete HP-50 | 118.87 |
| $m^3$ of concrete HP-55 | 123.64 |
| $m^3$ of concrete HP-60 | 128.41 |
| $m^3$ of concrete HP-70 | 137.95 |
| $m^3$ of concrete HP-80 | 147.49 |
| $m^3$ of concrete HP-90 | 157.02 |
| $m^3$ of concrete HP-100 | 166.56 |

including the cost of raw materials extraction, manufacture and transportation. The measurements ($m_i$) concerning the construction units depend on the design variables.

$$C = \sum_{i=1,r} p_i \times m_i(x_1, x_2, \ldots, x_n) \tag{6}$$

The structural constraints represented by equation 5 check the serviceability and ultimate limit states (SLS and ULS) and the geometrical and constructability requirements, following the Spanish codes for this type of structure (Ministerio de Fomento 2008, 2011) and the Eurocodes (European Committee for Standardisation 2005, 2003).

### 3.3 *Simulated annealing algorithm*

The heuristic algorithm used in this paper to carry out the optimization problem is the simulated annealing (SA) algorithm. SA was originally proposed by Kirkpatrick (1983) based on the analogy of crystal formation. SA algorithm generates an optimization process in which there is not a severe acceptance criterion, this is due the fact that the algorithm accepts worse solutions as long as a random number of uniform probability between 0 and 1 is lower than the probability expressed in the equation 7:

$$P = e^{\frac{E(\mu)}{T}} \tag{7}$$

where E is the difference between the objective function value of the current solution and the new solution, and T is the temperature that represent the cooling of the process. Therefore, new solutions that improve the objective function value are always accepted, while worse solutions have a probability of being accepted according to their aptitude and the temperature.

The temperature is the parameter that it is in charge of adjusting the number of acceptances. The initial temperature is calibrated following Medina's (2001) method, which proposes that initial temperature is halved when the percentage of acceptances is greater than 40%, and doubled when it is less than 20%. After that, the temperature decreases according to a coefficient of cooling $k$ following the equation $T = k{*}T$, when a Markov chain ends. The algorithm finishes after three Markov chains without improvement.

### 3.4 *Optimization process*

As described above, in this study, a comparison between two optimization processes will be carried out. Depending on the type of optimization the process and the design variables will be different. On the one hand, in the conventional heuristic optimization the bridge design is totally defined by the variables and the constraints that verify the limit states. On the other hand, the kriging-based heuristic optimization is more similar to standard method. That implies that only the cross section and concrete strength are variables from which the amount of the steel is calculated according to the constraints.

#### 3.4.1 *Conventional heuristic optimization*

In the conventional optimization, in addition to the seven geometrical variables and concrete strength, the reinforced steel and the prestressed steel are also variables.

Reinforcing steel is defined by 23 variables, 15 for the longitudinal reinforcement and eight for the transverse reinforcement (Figure 5). Longitudinal reinforcement is defined by the number of bars per meter and the diameter, placed at the top slab ($LR_{n1}$, $LRØ_1$), the flange ($LR_{n2}$, $LRØ_2$, $LR_{n3}$, $LRØ_3$), the web ($LR_{n4}$, $LRØ_4$), the bottom slab ($LR_{n5}$, $LRØ_5$) and the core ($LR_{n6}$, $LRØ_6$). Besides, extra bending reinforcement is divided into two systems. One covers the top slab at the support zone (L/5 on both sides of the piers), with a diameter defined by $LRØ_7$ and the same number of bars per meter as $LR_{n1}$. The other is placed at the bottom slab throughout the rest of the external span ($LRØ_8$) and central span ($LRØ_9$). The number of bars per meter is, for both locations, equal to $LR_{n5}$. The diameter can change among 0, 10, 12, 16, 20, 25 and 32 mm. Regarding transverse reinforcement, the diameter of the standard reinforcement ($TRØ_1$, $TRØ_2$, $TRØ_3$, $TRØ_4$, $TRØ_5$, $TRØ_6$, $TRØ_7$) is set with the same spacing ($TRS$).

Once the first pedestrian box-girder bridge is completely defined, SA algorithm makes movements of the design variables in each step, and a comparison between the objective function is carry out to finally obtain the cost-optimized pedestrian box-girder bridge according the process defined in the point 3.3. Each movement requires the complete verification of the SLS and ULS entailing a high computational cost.

#### 3.4.2 *Kriging-based heuristic optimization*

Kriging is a metamodel that has its origins in geostatic applications involving spatially and temporally

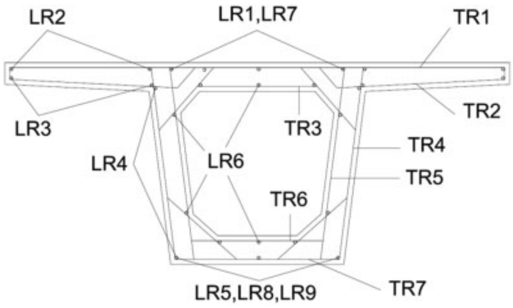


Figure 5. Reinforcing steel.

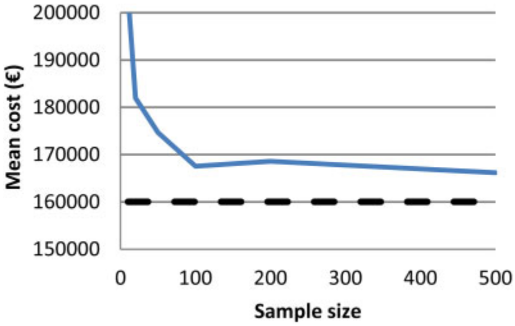


Figure 6. Comparison of the mean cost-optimized bridges.

correlated data. The kriging approach treats the objective function of interest as a realization of a random function (stochastic process) y(x). For this reason the mathematical model of kriging has been presented as a linear combination of a global model plus departures:

$$y(x) = f(x) + Z(x) \tag{8}$$

where $y(x)$ is the unknown response surface, $f(x)$ is the known (usually polynominal) function of $x$, and $Z(x)$ is a realization of a stochastic process with mean zero, variance $\sigma^2$, and non-zero covariance. The mathematical development of kriging are explained in Cressie (1990) and Simpson et al. (2004).

In contrast to the conventional heuristic optimization, in which the bridge is defined completely at the beginning of each iteration to later verify all the constraint defined by the codes, the kriging-based heuristic optimization only defines the design variables that the engineers would take into account in their design (geometrical variables and concrete strength) to later calculate the post-tensioned steel and the reinforced steel according to the codes.

First of all, a specific number of sample size ($N$) of data points belonging to the design space are obtained according to latin hypercube sampling, and afterword, the cost is obtained for each combination. Some of these points can be no-feasible due to geometric constraints. In these cases, the next condition is imposed: if the cost is higher than the minimum cost of the set of feasible solutions the cost considered is the real cost, otherwise, the cost is penalized. In this way, the response surface optimization will trend to feasible solutions.

Once obtained the response surface of kriging, a validation process that compares the real cost and the predicted cost of nine random data points is carried out in order to know the accuracy of the model. Finally, the heuristic optimization by means of SA algorithm is carried out to reach the best cost-optimized box-girder pedestrian bridge through the metamodel represented in Figure 2.

## 4 RESULTS

The comparison between conventional heuristic optimization and kriging-based heuristic optimization is shown in Figures 6–8. For this purpose, nine cost-optimized solutions have been obtained for each sample size (N). After that, the mean cost of these nine solutions has been obtained and a sensitivity analysis has been carried out for each sample size. Different N has been considered including 10, 20, 50, 100, 200, and 500.

For each case different characteristics of the two procedures followed to optimize the box-girder pedestrian bridge, have been compared. Figure 6 shows the mean cost of the nine cost-optimized solutions obtained by the different types of procedures. The horizontal dashed line represents the mean cost obtained by conventional heuristic optimization, while the solid line represents the mean cost obtained by the kriging-based heuristic optimization according to the different number of initial population. The mean cost of the conventional heuristic optimization is 160048.42 €. This cost is 3.84% lower than the best mean cost of the kriging-based heuristic optimization that correspond to N = 500. In addition, from N = 100 to N = 500 the ratio of improvement is 0.82%.

The computational cost is probably the main advantage of the use of metamodels-based heuristic optimization. Figure 7 shows the time spent by the different types of optimization. It should be indicated that the time considered by the kriging-based heuristic optimization take into account both generation of the initial population and heuristic optimization. As above, the horizontal dashed line represents the mean time spent by conventional heuristic optimization to achieve one cost-optimized solution, while the solid line represents the mean time spent by the kriging-based heuristic optimization according to the different number of sample size. The kriging-based heuristic optimization spends at most 1788.22 seconds when the response surface is generated by an initial population of 500. This time is 91.58% lower than the conventional heuristic optimization. Furthermore, if we accept an initial population of 100 (Figure 6), the time saving is the 97.9% respect to the heuristic conventional optimization.

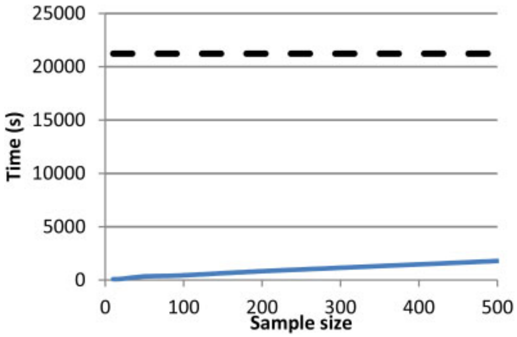


Figure 7. Comparison of the mean time spent.

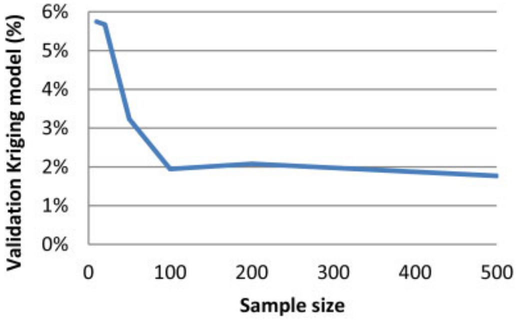


Figure 8. Accuracy of kriging model.

In addition, a comparison between the real cost and the cost predicted by the response surface of nine random solutions of the design space have been calculated to obtain the accuracy of the kriging model. Figure 8 shows that the precision of the kriging model increases with the initial sample size according to an horizontal convergence from the 5.35% of N = 10 to 1.85% of N = 500. That implies that a kriging model with a lower number of N achieves practically the same accuracy and reduces the computational cost.

## 5 CONCLUSIONS

Due to the high complexity of the structural optimization problems, heuristic or metaheuristic algorithms provide the greater efficiency in the conventional optimization. However, the computational cost of conventional optimization remains high. To solve this problem, surrogate models or metamodels can be used to reduce the computation cost of the optimization. One of the most encouraging metamodels used in the structural optimization is the kriging model. This is achieved generating a response surface based on a sample of data points in which the objective response is known. Latin hypercube is used to obtain the sample.

In this work, a comparison between conventional heuristic optimization and kriging-based heuristic optimization considering different sampling size is carried out for a concrete box-girder pedestrian bridge. The objective function is the cost. The results show that kriging-based heuristic optimization increases the mean cost of the conventional heuristic optimization of 3.84%. Besides, the results are stabilized by a sampling size of 100. Therefore, this sample is enough to obtain satisfying results.

This paper shows that the results obtained using metamodels are very close to the conventional heuristic optimization, with a significant saving of computational cost. This reduction in computational cost not only involves a faster optimum structural design, but can also provide an opportunity to perform processes that would be complicated by the conventional heuristic optimization due to the high computational cost, such as the optimum robust structure design.

## 6 FUTURE RESEARCH

It should be noted that, in order to evaluate the robustness of optimized structural designs, a faster process should be implemented to minimize the computational cost. The present paper serves as an illustration of the proposed way to achieve this reduction in computational cost. Kriging model can be applied to study the sensitivity of the optimum solution according to the variability of the objective function and the constraints. In this way, robust solutions could be obtained and minor variations of the optimum solution will not affect the integrity of the structure. Further development in this direction is undergoing.

## ACKNOWLEDGEMENT

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