



## Review

## Hybrid steel girders: Review, advantages and new horizons in research and applications

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## ARTICLE INFO

## Keywords:

State-of-the-art  
Hybrid girder  
Hybrid ratio  
Yield strength  
High-strength steel

## ABSTRACT

Although it is still common practice to use homogeneous steel girders (same yield strength in the flanges and web), implementing hybrid configurations seems to be an excellent alternative to improve the performance and sustainability of this type of structural element. Therefore, this paper provides a comprehensive review of the current state of knowledge on hybrid steel girders. The objective is to improve our understanding of this innovative and sustainable alternative to traditional homogeneous steel elements, with a focus on updating the theoretical basis for future design projects. The study analyzes 128 publications, from which information is extracted on five categorical variables, reflecting the current situation of hybrid elements. In addition to studying each variable separately and highlighting the most relevant research to date, a more in-depth statistical analysis is performed. It is based on simple correspondence analysis, which allows identifying the underlying relationships among the variables. Results summarize the design methods implemented to calculate these structures. Furthermore, the recommended hybrid ratios to achieve the best performance are presented. However, it is found that there are gaps in the research. Consequently, several promising lines of investigation are proposed.

## 1. Introduction

The growth and prosperity of nations have been closely tied to the ability to effectively connect different regions and build structures quickly, safely, and efficiently. Steel has played a crucial role in this development, starting with its use in constructing landmarks such as St. Anne's Cathedral in England (1772) and the Coalbrookdale Bridge (1779). The early 19th century saw further innovations, such as the Pont de les Arts in Paris (1801). Iconic structures such as the Brooklyn Bridge (1870) and the Eiffel Tower (1889) helped to establish steel as a material of choice for construction, driving the growth of the metal structures industry. The use of steel structures remains a crucial aspect of contemporary bridge and building construction due to its versatility, cost-effectiveness, and rapid construction time. Steel structures also offer design flexibility with the ability to create unique shapes using catalog elements or custom designs.

In the design and construction of steel structures, it is typical to use homogeneous type "I" beams with equal yield strength in both the web ( $f_{yw}$ ) and flanges ( $f_{yf}$ ). Steel can be classified into two categories based on yield strength: medium-strength steels ( $f_y = 235\text{--}460$  MPa) and high-strength steels ( $f_y > 460$  MPa), according to Girao Coelho et al. [1].

The development of high-strength steel (HSS) is an ongoing process driven by advancements in metallurgical technology, resulting in improved strength and performance specifications. HSS represents a practical solution for structures subjected to high stress and strain.

The trend toward hybrid construction materials is rising, including steel-concrete, concrete-high strength concrete, concrete-plastics (composites), and steel-special steels. Researchers are exploring ways to improve the sustainability of these structures by taking advantage of the benefits of using different materials in a structural assembly. A group of studies focused on the sustainable design of steel-concrete composite bridges, as reviewed by Martínez-Muñoz et al. [2], examines the design, maintenance, life cycle analysis (LCA), and decision-making aspects of this type of structure. For instance, Yepes et al. [3] optimize the design of a steel-concrete composite pedestrian bridge using heuristic techniques, while Martínez-Muñoz et al. [4] compare various concrete bridges to a steel-concrete composite variant using LCA. In Martínez-Muñoz et al. [5], the optimization of a bridge using swarm intelligence algorithms is performed, and in Martínez-Muñoz et al. [6], embodied energy is used as an additional optimization objective along with economical cost. These studies demonstrate hybrid materials' characterization, optimization, and decision-making application throughout their life cycle. Although

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the methodologies presented here focus on steel-concrete hybridization, they can be adapted to other materials.

Back to the topic of steel, according to Kulkarni and Gupta [7], in bending, a large portion of the stresses are absorbed by the flanges, which requires them to be thicker than the web. It often results in underutilization of the web material and leads to heavy and inefficient beams. Increasing the yield strength of the entire section can reduce thicknesses, but also increases the material cost. A more cost-effective solution is to use different types of steel in the flanges and web. This concept refers to hybrid steel elements. This approach optimizes material usage and offers a more rational solution in the construction industry. The concept of hybrid ratio (Rh) in girders refers to the ratio of the yield strength of the flanges to that of the web ( $f_{yf}/f_{yw}$ ). When the hybrid ratio differs from 1, the girder is considered a hybrid element, while a homogeneous girder has a hybrid ratio of 1. Using hybrid girders in construction leads to reduced weight, which translates to lower manufacturing, assembly, and construction costs. Additionally, hybrid girders offer a more sustainable solution with the potential for significant reductions in CO<sub>2</sub> emissions, aligning with the European Union's 2030 plan for achieving climate neutrality by 2050. Hence, the hybridization of materials, particularly steel-steel, provides a promising solution to address the challenges faced by the construction industry today.

Despite the potential benefits of hybrid steel elements, the utilization of these structures has been limited. The concept of hybrid steel elements dates back to the 1940s, and design methods have been established. However, there is a need to enhance their implementation in actual structures. Branches such as structural optimization or sustainability analysis offer opportunities for the rational design of hybrid steel elements, be it in their simple form as I-sections or in more complex systems such as hybrid box girders. This review aims to summarize the current state of knowledge on hybrid steel elements and provide a framework for future studies that can incorporate these structures in real-world structural engineering projects.

For the development of this review, the analysis of several categorical variables is proposed. Along with the bibliometric variables, we aim to examine the *research topics*, the *methodology* employed, the *experimental loading conditions*, the *yield strength* of both *the flanges* and *the web*, and the corresponding *hybrid ratios*. However, the analysis of this information from over 120 publications presents a wide range of results, making it difficult to generalize the findings. Therefore, a comprehensive statistical analysis is conducted to uncover the underlying relationships among the variables and consolidate the most critical insights from the studied information. In addition, practical considerations in design and construction are summarized for a better understanding of the use of hybrid girders.

The organization of this paper is as follows. Section 2 explains the general methodology followed in the research. Section 3 analyzes the information collected, beginning with the bibliometric analysis and then studying each categorical variable. Section 4 summarizes the most important practical considerations related to hybrid girders. Section 5 discusses the previously analyzed results, including statistical analysis to encompass knowledge. Finally, in section 6, the conclusions and promising lines of research are presented.

## 2. Methodology

The present review has been carried out in several phases and systematically, maximizing the number of publications and the information extracted from them. The research process followed the established guidelines described by Martínez-Muñoz et al. [2] and Martínez-Martín et al. [8].

The first stage consisted of selecting keywords for the topic to be addressed to perform a broad search of results in the Scopus and Web of Science databases. The search was limited to scientific and conference papers related to structural engineering. Some of the keywords and

combinations used were: *steel beam*, *hybrid beam*, *hybrid steel beam*, *high-strength steel beam*, *high-performance steel beam*, and *hybrid steel bridges*. Data such as authors, country of research, year and journal of publication, and categorical variables related to hybrid steel elements were extracted from retrieved publications.

The second stage of bibliographic research, known as “retrospective search”, involves selecting significant references from the previously gathered publications. In the following “prospective search” stage, a similar process is utilized, wherein the search is carried out among works that reference the selected publications from the retrospective search. This iterative process, combining keywords and “backward and forward” searching, resulted in identifying over 120 relevant publications. All collected information was consolidated into a database for further analysis.

For the analysis of the collected data, a visual representation was created using pie charts, bar graphs, and various diagrams that provide a clear and concise view of the information. Each categorical variable was analyzed separately, emphasizing the most prominent studies on each topic related to hybrid girders. It provides an initial understanding of the advancement in research for various related issues.

The final stage of the study focuses on discussing and analyzing the results. Simple correspondence analysis is utilized to examine the relationships between the categorical variables. This holistic approach, combined with the individual analysis of each variable in Section 3, provides a comprehensive understanding of the phenomenon being studied. The statistical analysis concludes the discussion, and the conclusions drawn from it together with those formulated when analyzing each variable separately, indicate the direction of future research.

## 3. Review results

The literature review results are divided into aspects of interest within hybrid girders. The bibliometric analysis is presented first, followed by a detailed examination of each categorical variable. The results are visually displayed, and the most important studies are emphasized.

### 3.1. Bibliometric analysis

Two of the data extracted from the documents is the year and country where the research was conducted. Table 1 presents the results of the recurrence of countries concerning the number of papers published. As can be seen, most of the research on hybrid beams has been carried out

**Table 1**  
Distribution of publications by country.

Country	Quantity	Reference
USA	37	[9–29] [30–45]
China	23	[46–66] [67,68]
Korea	12	[69–80]
India	11	[7,81–90]
Japan	9	[91–99]
Spain	7	[100–106]
Sweden	5	[107–111]
Australia	5	[112–116]
Canada	4	[117–120]
Czech R.	2	[121,122]
France	2	[123,124]
The Neth.	2	[125,126]
U.K.	2	[127,128]
Portugal	2	[1,129]
Egypt	1	[130]
Finland	1	[131]
Hungary	1	[132]
Iran	1	[133]
Poland	1	[134]
Singapore	1	[135]
Saudi A.	1	[136]

in the United States, which shows that the development of these elements has been of great importance in engineering in this country. In other Asian countries such as China, Korea, India, and Japan, the number of publications is also representative. Fig. 1 summarizes the information in Table 1. In addition, the countries were grouped by continent, showing a better picture of the concentration of studies. It can be seen that Asia leads in the number of studies, with 45%, followed by America, with 31%, and Europe, with 19%. In Africa and Oceania, only 5% of the research has been conducted, with five publications in Australia and one in Egypt.

Fig. 2 shows the distribution of published documents across different decades and continents. It provides insight into the evolution of the research and the growing interest in the topic in different regions. The study of hybrid girders began in the 1940s, with the initial definitions by Wilson [36]. During the following decade, however, research progress was slow. In the 1960s, research was reinvigorated with seven publications, including contributions by Natarajan and Toprac [33] and Fielding and Toprac [29] in fatigue. Additionally, Frost and Schilling [38] conducted research on the web's behavior under static loading, while Schilling [39,40] studied the flexural behavior of hybrid beams. Finally, Carskaddan [41] published work on shear buckling, and Haaijer [37] about the economic advantages of using HSS.

In the 1970s, research continued but was less active, with only four papers published. Some of the notable works from this period include Nethercot's study on post-buckling [127] Nagaraja et al.'s examination of hybrid columns [30], Toprac's compendium of flexural and shear tests on hybrid girders [28], and Maeda's investigation of the flexural behavior of these elements [58].

In the 1980s, the pace of research on hybrid girders remained steady, with five papers primarily focused on computational optimization and verification of beams. These papers included the work by Abuyounes and Adeli [20,44], who optimized hybrid beams through geometric programming, and Adeli and Khing [16], who performed computational comparisons of hybrid and homogenous elements. Other notable contributions at the time include Knight's study highlighting the economic advantages of hybrid beams [21] and Zahn's review of standards related to hybrid elements [25]. It is worth noting that all of this research was conducted in the United States, except for Maeda's work [72].

In the 1990s, research on hybrid girders experienced significant growth. It was conducted in multiple regions, including America, Europe, and Asia, with ten papers published on the subject. This decade saw essential contributions to the field, such as Dhillon and Kuo's work on optimizing the design of hybrid beams using geometric methods [14].

Adebar and Van Leeuwen's research on bridges fabricated with hybrid beams [119], Suzuki et al.'s examination of the effect of local web buckling in hybrid beams [92], and Ahlenius's study of hybrid beam behavior [111] were among the other notable contributions during this time.

With the start of the new millennium, research on hybrid and HSS experienced significant growth with the publication of 36 papers and the participation of authors from various countries. It led to advancements in various fields, such as the development and application of HSS, as presented by Bjorhovde [10], and the examination of economic aspects by Horton et al. [22]. Notable research also includes Azizinamini et al.'s studies on shear [11,12], Chacon et al.'s investigation of shear impact and patch loading resistance in hybrid beams in Europe [103], Veljkovic and Johansson's examination of hybrid beam design [109], and Ito et al.'s work on the rotation capacity of hybrid elements in Asia [93]. Tanaka et al. [135] proposed a method for designing stiffeners in hybrid beams. At the same time, Lateef et al. [130] conducted the only study on hybrid beams in Africa, exploring the behavior of beams under pure bending. The research in this decade highlights the advantages of using hybrid elements and provides several equations for their design.

In the last decade, research on hybrid beams has seen a marked increase, with 48 publications. While the number of studies has risen in Europe and Asia, the number of publications from America has remained limited, with only two papers. These include the work of Shokouhian et al. [9], who investigated the direct method for hybrid beam design based on slenderness, and Subramanian and White [35], who studied torsional buckling in uniform moment regimes. In Europe, Chacon et al. [102] continued their investigation into the behavior of hybrid beams under patch loading. Skoglund et al. [103,104] approached the topic of hybrid beams from an optimization perspective and reviewed bridge structures built with this type of beam. The authors reviewed hybrid bridges built in the USA, Japan, and Europe between 1990 and 2015. Another researcher delved into the study of hybrid beams was Juhas [121], who focused on fatigue. In Asia, notable studies included Ajeesh and Sreekumar's [85] evaluation of the shear behavior of hybrid beams and Sreekumar's [86] examination of the impact of imperfections on shear-subjected beams. Khatri et al. [87] economically compared the use of hybrid beams to conventional ones, while Ueda et al. [94] conducted a parametric study of flange buckling in hybrid beams.

Finally, in this decade, significant progress was made in the field of hybrid beams, particularly in Asia, with a strong emphasis on research conducted in China. Many researchers in this region dedicated their efforts to studying HSS beams, focusing mainly on their bending

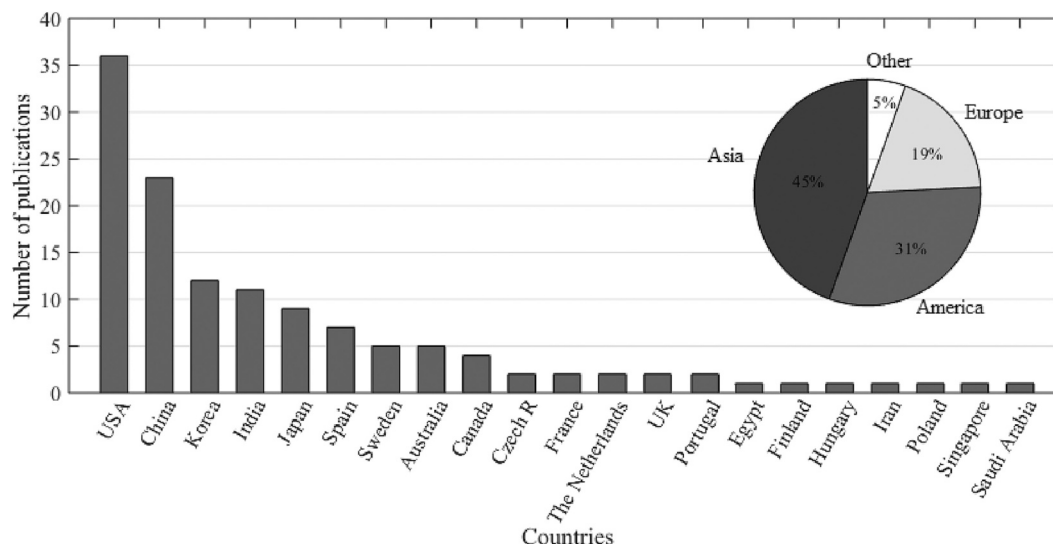


Fig. 1. Publications by country and continent.

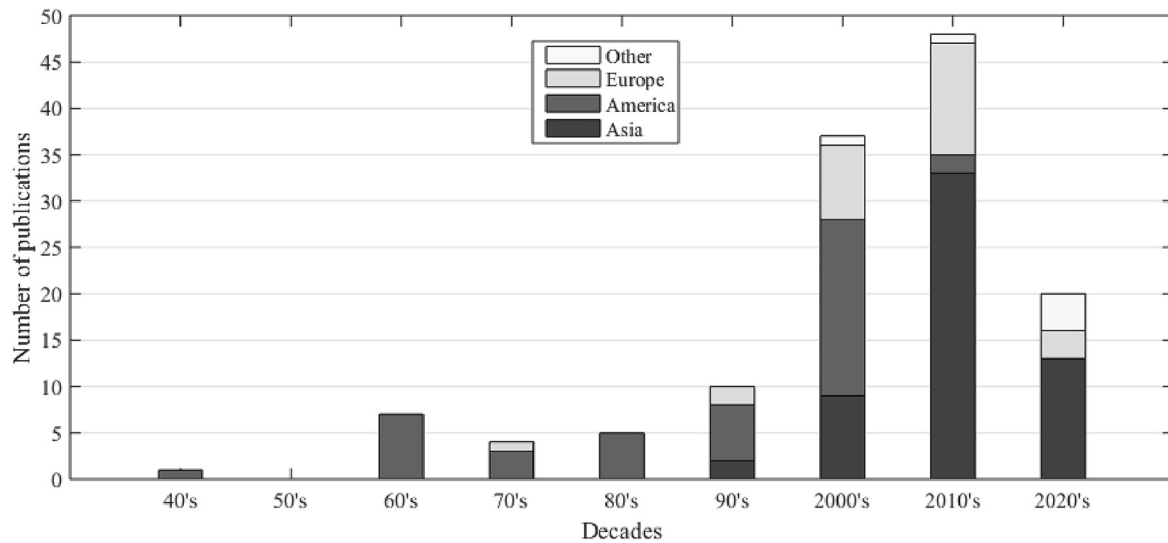


Fig. 2. Research by decade.

behavior [59,65,66] and incorporating aspects such as web openings [63] into their models. They also explored the impact of various types of buckling on the structural performance of these elements [60,62,64] and evaluated the structural behavior of hybrid beams under fire exposure [61]. It is worth mentioning the work of Bhat and Gupta [90] in India, where they analyzed the behavior of hybrid beams with closely spaced web openings. In general, the rapid advancement in this area can be attributed to the growth in the construction industry in Asia, particularly China.

In addition to the regional and chronological analysis, it is helpful to

know the journals of the papers' publication. Fig. 3 shows how the publications are distributed among the different journals, with a significant predominance of the Journal of Constructional Steel Research over others, such as Engineering Structures or Journal of Structural Engineering, which also present a considerable number of publications related to the subject in question.

### 3.2. Research topics

Regarding the research on hybrid elements, various topics have been explored to understand their behavior and benefits better. As depicted in Fig. 4, most studies focus on the analysis of hybrid beams subjected to pure bending, pure shear, or a combination of both.

#### 3.2.1. Bending

In the case of stresses produced by pure bending, 36% of the papers dealt with this topic. The slenderness of the elements plays a crucial role in the analysis of this phenomenon, as it influences the development of plastic hinges and, ultimately, the ductility of the element. This consideration is particularly relevant in the case of hybrid beams, as their use is aimed at reducing thicknesses and weight, which in turn reduces costs. Thus, hybrid beams can have the same dimensions as their homogeneous counterparts but with lower thicknesses.

It is usual for hybrid beams to have a non-compact web and flanges, which corresponds to class 4 in Europe. In their research, Lateef et al. [130] recommend a  $h_w/t_w$  value of 120, with which the cost-benefit ratio is well balanced. Another aspect on which the different investigations agree is that the bending capacity is significantly affected by the amount and distance between the flanges' lateral bracings and the web stiffeners. These bracings should limit the occurrence of local or torsional buckling failures in the elements, as expressed in the paper written by Wang et al. [50]. This paper, in which different bridges in Japan are reviewed, concludes that a value  $\lambda_{LT} = 2.3$  is an optimum value for the slenderness of the beam. This slenderness is given by the Eq. 1.

$$\lambda_{LT} = \sqrt{W_y F_y / M_{cr,LTB}} \tag{1}$$

Here,  $W_y$  is the section modulus of the beam,  $F_y$  is de yield strength, and  $M_{cr,LTB}$  is the critical moment of the section concerning lateral torsional buckling, which can be obtained as in Eq. 2.

$$M_{cr,LT} = \frac{\pi}{L} \sqrt{EI_y GJ} \left( 1 + \frac{\pi^2 EC_w}{L^2 GJ} \right) \tag{2}$$

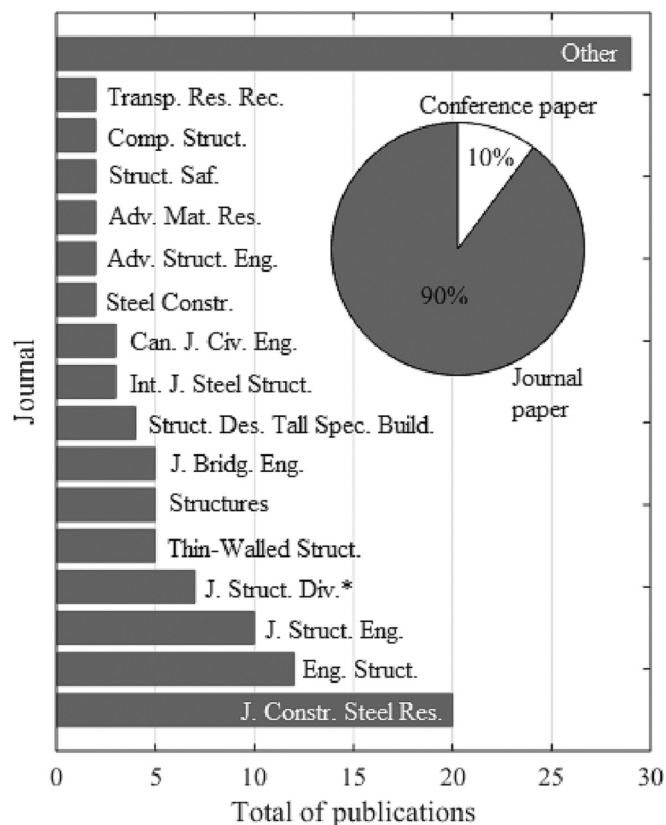


Fig. 3. Distribution of publications by journals. \* Journal of the Structural Division (ASCE) was active from 1956 to 1982.

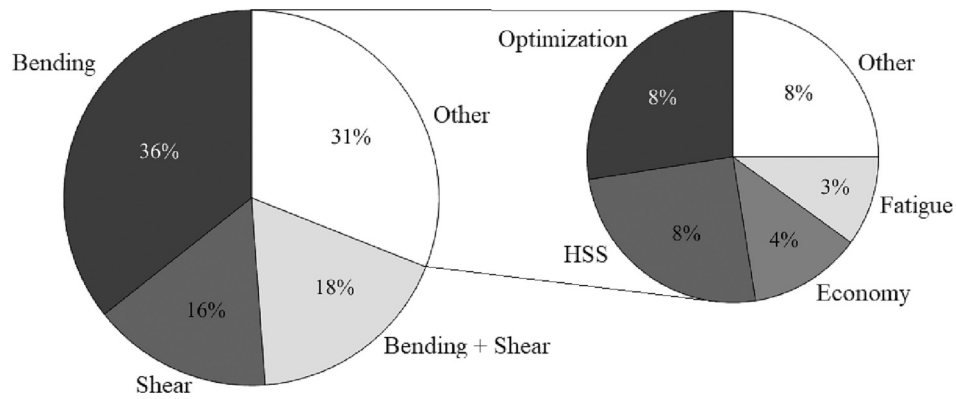


Fig. 4. Research topics.

Where  $L$  is the unbraced distance,  $I_y$  is the inertia in the weak axis direction,  $J$  is the torsional modulus, and  $C_w$  is the warping modulus of the section.  $E$  and  $G$  are the elastic and shear moduli of the steel, respectively. In Wang et al. [50], it is also concluded that it is ineffective, in the flexural design of a beam, to increase the web thickness since the flanges are the primary stress-resisting elements.

In their computational study, Kulkarni and Gupta [7] concluded that the optimal hybrid ratio is 1.4. This ratio ensures efficient utilization of the enhanced strength of the flanges, preventing the web from reaching high-stress levels that could result in buckling. It is noteworthy to mention that this finding is not meant to be a normative limit.

### 3.2.2. Shear

Another topic of focus in the studies reviewed is shear, which is addressed in 16% of the papers. These studies concentrate on analyzing and comprehending the behavior of the web after buckling and its potential to generate a diagonal stress field as a mechanism of resistance. Unlike the section bending analysis, where buckling of the section was seen as unfavorable, in shear analysis, web buckling is crucial for the strength of the beam.

Azizinamini et al. conducted one of the shear studies [11]. They addressed the limitation imposed by the AASHTO standards on utilizing the development of the diagonal stress field in hybrid beams, which was previously allowed in homogeneous beams prior to 2003. The study provided the opportunity to incorporate the stress field in hybrid beams, overcoming a significant hindrance in their practical use. This limitation, which reduced the shear capacity of hybrid beams compared to

homogeneous ones and impacted their economic feasibility, was thus alleviated. For the study, the authors designed, fabricated, and constructed eight beams, six of which were hybrid. Thus, they verified the specimens' post-shear behavior and compared the performance of hybrid and homogeneous members in the inelastic range. They found that, in general, the beams, regardless of their hybrid ratio, exhibited higher strength than what was predicted using the standard's equations. The graph in Fig. 5, created from the authors' data, highlights the increase in strength observed when the diagonal stress field was taken into account in the predictions made using the AASHTO standard, which was confirmed by the tests conducted on the various beams. The values shown for the three series are the expected (in the case of the first two series) and actual strength values of the specimens compared to the presumed strength of the elements calculated using the standard, which disallows the inclusion of the effect of the tensile field in hybrid sections. The "TFA" series ignores the limitation on tension field action and includes that contribution for all specimens. The "R = 1" series eliminates the reduction due to shear moment interaction, which is equivalent to taking the reduction factor as equal to 1. Finally, the "Real test" series shows the results obtained from the failure of the elements in the laboratory. According to this figure, if the diagonal stress field is considered, substantial increases in section resistance can be expected. On the other hand, it shows coherence between the test results and what is expected according to the standards. For specimens 7 and 8, it is observed that their results are close to the horizontal axis since they are the homogeneous beams included in this study. For these specimens, the development of the stress field is allowed according to the standard.

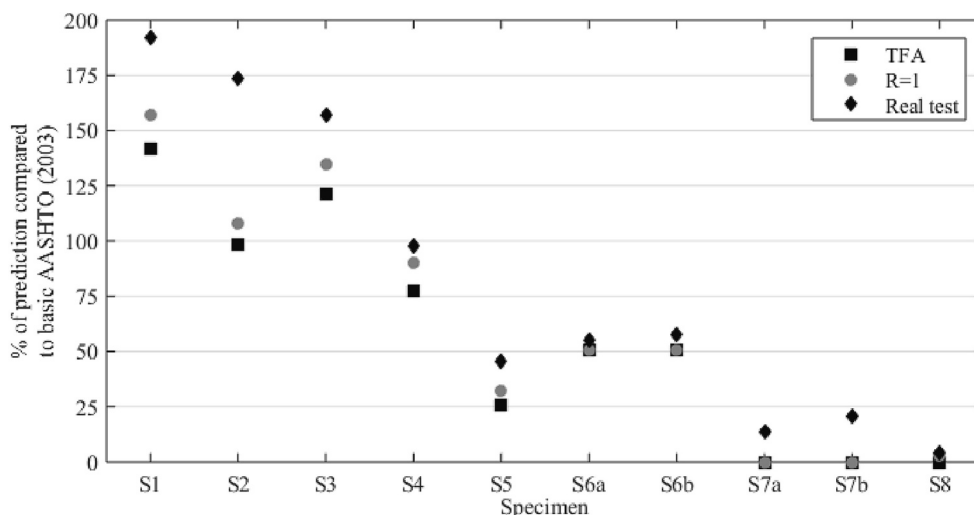


Fig. 5. Graphical representation of results obtained by Azizinamini et al. [11].

Therefore, in the initial prediction, this development was already included. Thus, it was demonstrated that there is no reason to have a restriction on considering the development of the diagonal stress field in hybrid beams.

Another relevant research is presented by Ahmad and Zoubi [17]. This study considers that until the publication of the AASHTO-LRFD 2004 standard, the use of the stress field as a contributor to the shear strength in hybrid beams was not allowed. Therefore, the structural design of the Blannerhasset Bridge across the Ohio River projected with the AASHTO LRFD 1998 and 2004 standards is compared. One of the conclusions obtained from their comparison was that, according to the standard used in 2003, 120 stiffeners were required along each main beam to avoid buckling. In contrast, according to the standard published in 2004, only 36 stiffeners were needed. It allowed economic savings of up to 10% on each beam simply by allowing the stress field to be included as part of the section resistance.

White and Barker [45] compared 12 models aimed at determining the model that demonstrates the highest accuracy in predicting the ultimate shear of a section while prioritizing simplicity in calculations. Their analysis was based on 129 tests, including 30 hybrid and 11 curved beams. In their findings, the authors identified three key factors that contribute to the shear strength of the section: the pre-buckling strength, the post-buckling strength, and the formation of Vierendeel frames between the flanges and stiffeners. With their analysis, the authors conclude that the most accurate model for predicting the ultimate shear of a section is the Bassler model, which is widely accepted by the AASHTO and AISC standards. This model assumes that the flanges do not fully anchor the web, resulting in the partial development of the stress field in the early stages of post-buckling. However, the equations proposed in the model fully capture the development of the stress field. The shear resistance of the section is determined by the combination of its pre-buckling shear resistance, its buckling-induced shear resistance, and the contribution from the development of the diagonal stress field.

In their paper, Ajeesh and Sreekumar [85] present a parametric study using finite element models to verify the post-buckling behavior. The results obtained were compared concerning the Cardiff, Bassler, and Lee & Yoo models. The parametric study analyzed the relationship between the behavior and the aspect ratio ( $d/D$ ), slenderness ( $D/t_w$ ), and web yield strength ( $f_y$ ). The results showed that for aspect ratios ranging from 0.75 to 2, the Lee & Yoo model produced the best match with the computationally obtained ultimate strengths. Meanwhile, the Bassler and Cardiff model had a good correlation with the results for aspect ratios ranging from 1.5 to 2. Other conclusions obtained from the study are that for aspect ratios between 1 and 2, and regardless of the yield strength of the web, the post-buckling resistance will be maximum and contribute between 50% and 60% of the ultimate shear. For other ratios, the material reaches the yield state before developing the stress field ( $d/D < 1$ ) or does not reach the stress field due to exaggerated deformation of the cross-section ( $d/D > 2$ ). Concerning the slenderness of the section, a higher percentage contribution of post-buckling to the ultimate strength of the section is achieved for a slenderness value close to 140, independent of the yield strength of the section. The slenderness of the panel influences the development of the stress field. The stress field starts to develop when the slenderness is low, meaning that the web is close to the yield point. On the other hand, when the slenderness is high, and the cross-section experiences excessive deformations, the full development of the stress field is limited, thus reducing its contribution to the resistance.

### 3.2.3. Bending-shear interaction

The last main topic to discuss is the interaction between bending and shear. It is the most common situation in beams since, as known from the materials' strength, shear and bending moments are closely and mathematically related.

Vejkovic and Johansson [109] conducted a comprehensive study that analyzed tests on hybrid beams performed by various researchers.

The aim of their research was to determine if the equations in Eurocode 3 could be applied uniformly to both hybrid and homogeneous beams. The results of their investigation indicated that the equations could be used without modification, and it was concluded that the early creep of the web can be ignored for bending strength.

Shokouhian and Shi [48] researched the different types of buckling failure in hybrid beams, whether it occurs by bending, shear, or a combination of the effects. Thus, they classified six failure modes. LCB is the lateral buckling, LTB is the lateral-torsional buckling, and SHB is the shear buckling. In addition, combined failure modes such as LCB + LTB, LCB + SHB, and LTB + SHB are possible, corresponding to the interaction. One of the study's main conclusions is that the combination of multiple failure modes influences the ultimate strength of the section. In particular, the dominant factor is lateral-torsional buckling. Another important conclusion is that interaction must be considered in the design of hybrid beams, as failure modes related to interaction significantly decrease the section's bending capacity.

On the other hand, Kovesdi et al. [132] went a step further in their analysis and included the scenario where a punctual load is applied, resulting in a transverse force ( $F$ ). This situation is commonly encountered during bridge construction, particularly when using launched beams, and it can lead to critical stress conditions if not adequately considered in the structural analysis. Before 2003, the regulations, including the European standards, lacked a straightforward method for addressing the interaction of bending, shear, and this transverse force. To address this, Kovesdi et al. conducted a parametric study to develop an equation defining the interaction between these three forces, as shown in Eq. 3.

$$\left(\frac{M}{M_{pl,R}}\right)^{3.6} + \left(\frac{V - 0.5F}{V_R}\right)^{1.6} + \left(\frac{F}{F_R}\right) = 1 \quad (3)$$

Here,  $M$ ,  $V$  and  $F$  are the moment, shear and transverse force acting on the loaded panel, respectively.  $M_{pl,R}$  is the design plastic resistance of the cross-section consisting of the effective area of the flanges and the fully effective web irrespective of its section class.  $V_R$  is the shear buckling resistance of the loaded web panel, and  $F_R$  is the patch loading resistance of the loaded web panel. This equation can be represented graphically as a piece of a sphere of unit radius between the positive  $X$ ,  $Y$ , and  $Z$  axes. Any point combination of  $M$ ,  $V$ , and  $F$  whose evaluation in Eq. 3 is  $< 1.0$ , i.e., within the limits of the sphere described above, will not exceed the structural capacity of the element.

Finally, Ghadami and Broujerdian [133] studied the interaction between bending and shear forces, considering a high-temperature condition. They proposed equations to calculate the interaction that did not consider shear buckling. They compared their results to six existing experiments to validate these equations and conducted a finite element analysis of 60 beams. Their findings are applicable to all types of beams, regardless of temperature exposure. It is valid if their failure is not associated with elastic or inelastic buckling in the web or flanges. The study's results can accurately predict the interaction phenomena compared to both laboratory tests and finite element analyses. Additionally, a factor was introduced to account for the degradation of the material's strength with rising temperatures by modifying the yield strength of the material as described in Eq. 4.

$$F_{y,T} = \sqrt{K_{y,T} K_{E,T}} F_y \quad (4)$$

Where  $K_{y,T}$  and  $K_{E,T}$  are the reduction of yield stress and Young's modulus at high temperatures, respectively.

### 3.2.4. Other topics

This chapter also discusses other topics that further clarify the hybrid beam landscape. Natajara and Toprac [28] investigated the performance of four beams with aspect ratios of 0.5 and 1.5 and stress levels ranging from 172 to 275 MPa (25–40 ksi) for one pair of beams and from 172 to 345 MPa (25–50 ksi) for another pair of elements. These beams

underwent initial static testing followed by fatigue-related studies as described in Fielding and Toprac [29]. The specimens subjected to lower stress levels (172–275 MPa) did not exhibit any cracks and could withstand many cycles without compromising stability. However, the specimens subjected to higher stress levels (172–345 MPa) displayed cracks, particularly the one with an aspect ratio of 1.5, which showed four cracks and was limited to 360,000 cycles. To better understand the different types of failure, the authors categorized cracks and their behavior into five categories. They also established limitations on the slenderness ( $h_w/t_w$ ) of the elements based on the results from over thirty specimens from previous studies and the four in their study. A slenderness limit that ensures the material will not crack over a cycle range of 500,000 to 2,000,000,000 cycles is determined using Eq. 5.

$$\beta_{lim} = 100 \sqrt{\frac{100}{f_{yf}}} \tag{5}$$

Here,  $f_{yf}$  is in Ksi. Otherwise, to guarantee a minimum of 100,000 cycles, Eq. 6 should be used.

$$\beta_{lim} = 150 \sqrt{\frac{100}{f_{yf}}} \tag{6}$$

Furthermore, Fielding and Toprac [29] conducted a study to examine the failure of three hybrid beams and to investigate the emergence of the diagonal stress field under conditions of interaction with bending moments. Rather than focusing on ultimate stresses, this investigation aimed to assess the fatigue performance of the section. The results showed that the beams with a high interaction index had a reduced life under cyclic loading, compared to those primarily subjected to separate bending and shear stresses.

Felkel and Rizos [43] studied the performance of bridge girders made of high-performance steels with a yield strength of 482 MPa (70 Ksi). They performed in situ measurements on a bridge in Gaffney, South Carolina that utilized HPS 70 steel in its negative moment regions. They also analyzed three half-scale specimens with varying bracing and slenderness conditions. The results were used to evaluate fatigue and structural performance, calibrate computational models for further parametric studies, and evaluate alternative designs. The study concluded that hybrid beams only experienced a loss of 0.5% to 8% of their bending capacity compared to homogeneous beams made of HPS 70 Ksi steel, demonstrating the benefits of having HSS in the flanges using lower yield strength steels in the web. The authors found no differences in fatigue results between conventional steels and the 70 Ksi steel tested. They emphasized the impact of imperfections and residual stresses in the material on the strength of the elements, as these could trigger premature or stability failures that significantly reduce their strength.

In a different aspect of the field, Abuyounes and Adeli [20] utilized the General Geometric Programming (GGP) algorithm to optimize the design of steel hybrid beams. Their study aimed to find a beam that satisfies specific criteria: (a) uniform section, (b) arbitrary loading conditions, (c) with or without stiffeners, (d) paired or single stiffeners, and (e) and laterally supported or unsupported. The application of the algorithm demonstrated the efficacy of the geometric approach for optimizing both stiffened and unstiffened hybrid beams, with the added benefit of being robust and straightforward when assuming uniform spacing between stiffeners. A similar study was carried out by Dhillon and Kuo [14], who presented an optimization process using the GGP method to optimize both stiffened and unstiffened beams. The method was based on the AASHTO load factor method. The objective was to minimize the weight of the beam.

Using an optimization approach, Mela and Heinisuo [131] conducted a study to compare the performance of HSS with yield strengths of 500 MPa and 700 MPa to conventional strength steel (355 MPa) based on the Eurocode 3 standard. The production, transportation, and

assembly costs were incorporated into the objective function, and constraints such as flexural and shear strength, lateral buckling, and displacement were included in the optimization process. The effects of torsional buckling were omitted in this study. The authors used Particle Swarm Optimization, a stochastic heuristic algorithm, to minimize beam weight. The results showed that the 700 MPa homogeneous beam was the lightest (beam 6), but the weight savings compared to homogeneous beams of 500 and 355 MPa were only 2% to 10% for distributed loads of 20kN/m and varying bridge spans. When considering cost optimization, hybrid beams proved to be the most beneficial solution, as they offered a potential reduction of up to 30% in costs, as seen in Table 2. The authors demonstrated that as the loads decrease, the weight difference between HSS and conventional steel decreases until it becomes negligible, but hybrid beams remain a cost-effective option.

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**Table 2**  
Summary of results obtained in Mela and Heinisuo [131].

Beam	$F_{yf}$	$F_{yw}$	$F_{yb}$	Opt Costs	Opt weights
	MPa			Dollars	Kg
1	355	355	355	524	410
2	500	355	500	498	352
3	500	500	500	538	344
4	700	355	700	486	318
5	700	500	700	530	304
6	700	700	700	555	302
7	355	355	500	538	410
8	355	355	700	555	410
9	500	355	355	510	382
10	700	355	355	490	342
11	500	355	700	512	352
12	500	500	700	555	344
13	700	355	500	490	330
14	700	500	500	538	322

Note:  $F_{yf}$  and  $F_{yb}$  are the yield strength in top and bottom flanges.  $F_{yw}$  is related to the web.

potential reduction of up to 30% in costs, as seen in Table 2. The authors demonstrated that as the loads decrease, the weight difference between HSS and conventional steel decreases until it becomes negligible, but hybrid beams remain a cost-effective option.

Regarding economics, several papers have demonstrated the advantages of hybrid beams over homogeneous ones. Price [21], in his research, evaluates each of the different aspects to be addressed in the design of a bridge and how each of them affects the final results structurally and economically. In general, it can be seen that, economically, hybrid beams have a significant advantage over homogeneous ones, except in the case of beams subjected to high shear forces. However, this is only because, at the time of the paper's publication (1984), the regulations restricted the use of the diagonal stress field in hybrid beams, which made homogeneous beams more competitive.

Barker and Schrage [15], taking as an example several bridges manufactured in the states of Nebraska, Tennessee, and New York, proposed alternatives to obtain more economical bridges. They designed bridges with 9, 8, and 7 main girders and modified their yield strength. It was concluded that the use of hybrid girders has a reduction of up to 60% in fabrication costs, 20% in erection, and 35% in the final cost of the bridge compared to other solutions.

### 3.3. Research method

The second categorical variable is related to the method used to develop the research. Four categories have been established: computational, experimental, a combination of both (either computational and experimental validation or vice versa), and research based on reviews. Fig. 6 displays the distribution of the research method used in each publication. Most publications rely on computer simulations, accounting for 31% of the total. It is followed by a combination of computational and experimental methods, which comprise 28% of the total. Research carried out by reviewing standards or other authors' experiments represents 22% of the total. Finally, experimental research only makes up 19% of the analyzed studies.

The prevalence of computer-based research highlights its cost-effectiveness and quick data analysis capabilities. These simulations result in a reliable output, but the validity of these results must be confirmed through experiments carried out in structural laboratories. Such experiments bring about a deeper understanding of the observed phenomena and can reveal unexpected behaviors. However, experimentation is a costly process that requires specialized facilities, precise measurements, and a significant amount of time for design, assembly, and data collection. Despite the challenges, experimentation remains an integral part of structural research. On the other hand, combining experimental and computational methods in the same study enables the

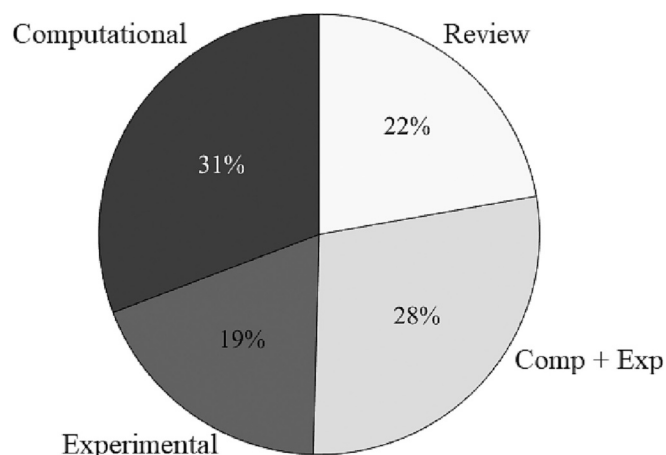


Fig. 6. Research methods.

validation of models and a broader scope of investigation through a parametric study beyond what can be achieved through specimen failure alone. This type of research provides the most insightful results on the topic and, although not the most prevalent methodology, has been gaining significant recognition in recent years. Table 3 summarizes the research method used by each author.

### 3.4. Loading condition for experimentation

According to the case studies reviewed in the literature, there are three ways of loading the specimens. The 3P condition creates maximum bending and shear at the point of load application, making it ideal for shear studies. The 4P condition, on the other hand, generates a constant moment span in the beam, making it optimal for analyzing the bending behavior because of the neutral shear span. Finally, uniform loading is used in other studies, such as optimization or cost-benefit analysis, where the aim is to gain a general understanding of the element rather than to specifically analyze its behavior under a particular type of stress or stress combination at a certain point.

Fig. 7 shows each loading configuration and the corresponding internal forces. It is important to note that in most of the cases analyzed, beams are simply bi-supported since in general, the interest of the investigation is based on a local objective of understanding the behavior of the section under shear and bending stresses. Thus, this analysis is independent of whether the moment or shear is negative or positive.

As previously stated, the most significant publications focus on the study of bending in beams. In Table 4, it can be appreciated that most of the publications use 4P-type loading tests. It corresponds to the previous comment on the use of this loading system for the study of pure bending. Accordingly, the predominant loading condition for the shear study is the 3P. For the study of the combination of bending and shear, there is no clear predominance of one configuration over the others. In other studies, there is an equal distribution among the different configurations. It is because within this category are grouped studies of varying nature, e.g., different types of buckling or the analysis of HSS beams embedded in other materials. In this section, the presence of a uniformly distributed load is mainly related to economic or optimization-related analyses. In Fig. 8, the distribution can be more readily appreciated. Here, N.S. refers to papers in which the loading condition is not explicit. It has more to do with research based on review approaches.

### 3.5. Yield strength of flanges and web

In this research, another noteworthy aspect is utilizing HSS in the structural elements under examination. Most of the elements used in the studies, as depicted in Fig. 9, have a yield strength between 200 and 500 MPa. As for the web, it is found that the range between 200 and 300 MPa is the most used. In the case of flanges, the most commonly used steel is in the range between 400 and 500 MPa. It is consistent with the typical arrangement in hybrid beams with lower yield strength in the web compared to the flanges. It is worth mentioning that there has been a rising trend in recent years toward research on steel elements with a yield strength >600 MPa.

It is noteworthy that some of the studies analyzed in this research feature asymmetric beams, either due to differences in yield strength ( $f_{y\uparrow} < f_{y\downarrow}$ ) or flange dimensions ( $b_{f\uparrow} < b_{f\downarrow}$ ). It is only observed in investigations that use mixed beams, where a top concrete slab reinforces the compression flange. Combining the two elements increases compressive strength and provides lateral stability to the compression flange. This design approach enables the use of the same yield strength in the web or a reduction in the size of the top flange, ultimately leading to a more efficient and cost-effective design of the steel beam. Hybrid beams used as composite elements have shown exceptional performance, and the equations used for homogeneous beams can also be applied, as demonstrated by Hendawi and Frangopol [23].



**Table 3**  
Research method used by publication.

Research method	References
Computational	[9,13,14,16,17,20,46–48,53,67,69,75,76,81–83,85–87,90,94] [102–104,108,112,118,119,125,126,130–134]
Experimental	[1,11,28,29,33,49,50,55,56,58,59,65,76,78,93,96,98,106,110,117,121] [122]
Computational + Experimental	[7,30,38,39,41,43,54,57–61,63–66,68,78,79,92,95,99] [100,113–116,123,127–129,135,136]
Review	[10,12,15,21,23–25,27,31,32,34,36,37,40,77,80,84,91,97,101,105] [107,109,111,120,124]

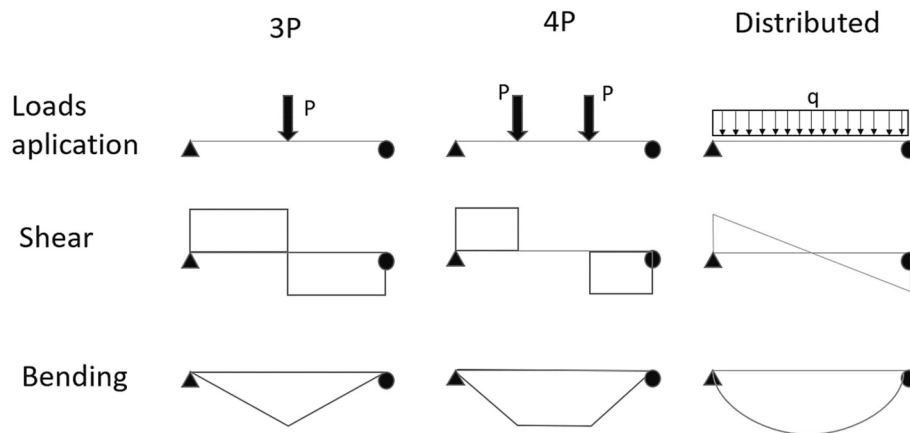


Fig. 7. Types of loads and corresponding internal forces.

**Table 4**  
Loading condition in function of the research topic.

Research topic	Loading condition	References
Bending	3P	[30,55–57,68,73,74,82,92,128]
	4P	[9,35,48,49,51,65,66,83,88–90,94,95,98,110,118,130,136]
	3P + 4P	[7,32,50,59,113]
	Uniform	[87,112]
Shear	3P	[11,13,81,85,86,96,100,105]
	4P	[1,119]
	3P + 4P	–
Bend + Shear	Uniform	[17]
	3P	[63,102–104,122,132,133]
	4P	[33,47,53,72,117,134]
	3P + 4P	[28,54,135]
Other	Uniform	–
	3P	[43,61,62,64,68,123]
	4P	[29,60,115,116,121,125,129]
	3P + 4P	[67,114]
	Uniform	[16,20,22–24,44,126,131]

### 3.6. Hybrid ratio ranges

The central focus of this research is the application of hybrid steel elements. The various documents reveal that the range of hybrid ratios studied is between 1 and 2. It is a slight predominance between 1.31 and 1.60, with an occurrence of 19%. As depicted in Fig. 10, this range has been determined to be optimal for beam design, according to several studies. If the  $R_h$  value exceeds 2, it becomes challenging to maximize the capacity of the flanges before the web fails. On the other hand, for  $R_h$  values lower than 1.4, the overstress mechanisms will not develop. The element will simply behave as a homogeneous element due to the minimal difference in yield strength between the flanges and the web. The high incidence of  $R_h = 1$  (homogeneous) beams is because many studies compare the behavior of hybrid beams with homogeneous ones. Numerous publications are also concerned with applying HSS, where homogeneous specimens are used to study specific phenomena such as

buckling, fire behavior, or composite elements (e.g., HSS beams embedded in cementitious materials).

The study conducted by Toprac [28] explored the behavior of hybrid elements with a ratio of 2.6 (featuring a yield strength of 250 MPa in the web and 690 MPa in the flanges). The research found that the stress redistribution in the beam occurs through the formation of a diagonal stress field. It is also highlighted the need for stiffeners to prevent local web failures increases as the hybrid ratio increases. Additionally, the author notes that the methods proposed by various standards for the design of homogeneous beams can be applied to hybrid beams.

In their research, Suzuki et al. [92] compare the  $M-\theta$  ( $M$  is the bending moment and  $\theta$  is the rotation angle) plots of hybrid beams concerning their homogeneous counterparts. It is also considered a homogeneous beam with a hybridization coefficient of  $<1.0$ . This study verifies that the best behavior is that of the hybrid beam with yield strength in the web of 400 MPa and flanges of 500 MPa, followed by the

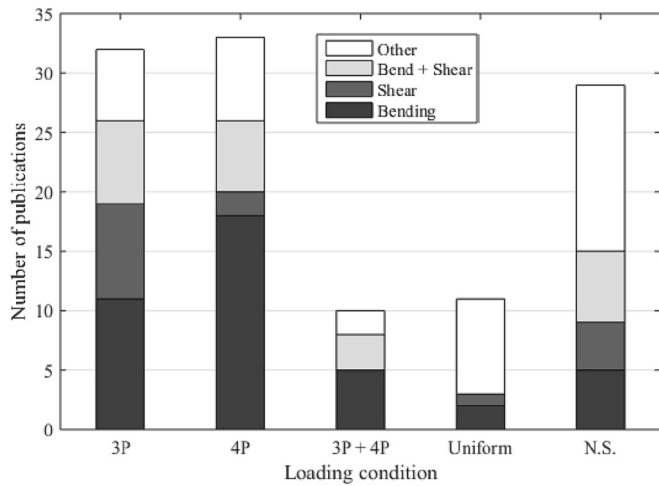


Fig. 8. Graphical representation of results shown in Table 4

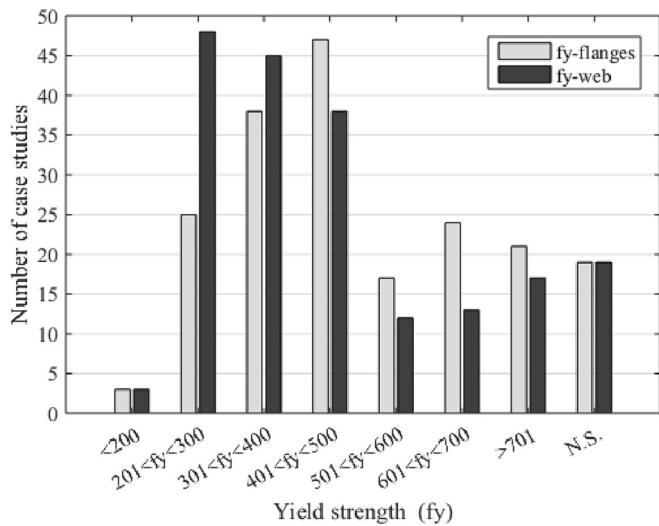


Fig. 9. Yield strength on flanges and web by intervals.

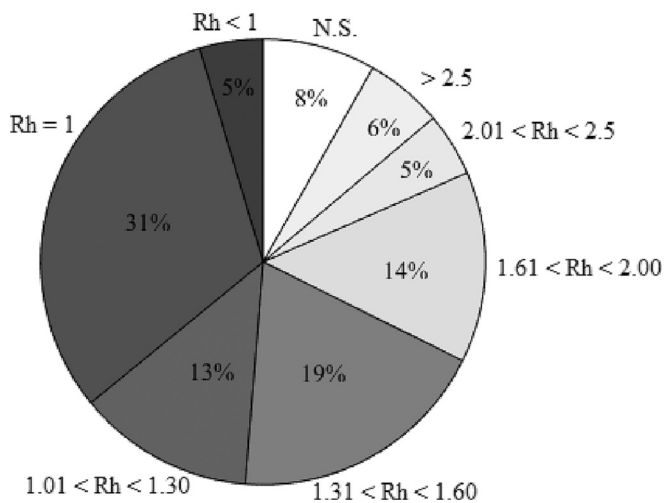


Fig. 10. Hybrid ratio (Rh) ranges according to the review.

homogeneous beams of 400 MPa and 500 MPa, respectively. The hybrid beam, with a web yield strength of 500 MPa and flange yield strength of 400 MPa, does not attain its section’s yield moment. It is because well-structured hybrid beams can attain significantly higher elastic limits than the beam’s first fiber. Regarding the comparison of homogeneous beams, the 500 MPa beam shows inferior behavior because as the yield strength increases, the material becomes more susceptible to various types of buckling. On the other hand, for beams with a hybrid ratio lower than 1.0, the stress field anchored at corners in a material with flange yield strength lower than the web results in premature specimen failure. Consequently, Suzuki’s paper highlights that hybrid beams with high-strength flanges and medium-strength webs exhibit improved behavior in the M-θ plot compared to homogeneous HSS beams. Additionally, as previously mentioned, as the yield strength increases, the rotational restraint of the beam becomes weaker due to a heightened risk of instability.

#### 4. Practical considerations for the design and construction of hybrid girders

This section discusses practical aspects related to hybrid girders. First, a summary of how their design is approached in the most significant codes is done. Finally, some practical considerations for their construction are presented.

##### 4.1. Design of hybrid steel girders according to standards

Once the bibliography has been analyzed, it is interesting to summarize the primary considerations for designing hybrid steel beams. It is essential to point out that there still needs to be more information in the standards on the design of this type of element. However, some make direct reference to the possibility of designing hybrid sections. One example is the EN1993-1-5 code, which “explicitly permits using different steel grades in flanges and webs in so-called hybrid girders. No detailed rules are given for designing such girders, but in all design rules, a subscript *f* for flanges and *w* for web indicate the relevant yield strength” [137]. Using the publication of Veljkovic and Johansson [109], which uses the code above as a basis but is enriched by other publications to update some aspects, the primary considerations for designing these elements are summarized.

**Determination of cross-section class:** The traditional way determines the flanges cross-section class (e.g., according to [138]). However, the web cross-section class should be determined using the yield strength of the compression flange. It is slightly conservative because the cross-section class is influenced by both stress and strain. Thus, only the strains in the web will correspond to those in the flange, not the stresses. It should be noted that hybrid beams generally have a Class 4 cross-section according to Eurocode 3.

**Bending resistance:** The difference in yield strength between the flanges and the web influences flexural strength. As mentioned above, usually, the web will yield partially before the flanges reach their yield strength. The influence of this phenomenon is different for diverse cross-section classes. The proposed formulas give the characteristic resistance, which must be divided with the appropriate partial safety factor. They will only apply to doubly symmetric I-beams. The same principles are used for other types of beams (monosymmetric, composite) but with variations. These can be found in [40].

##### 4.1.1. Cross-section classes 1 and 2

The bending strength  $M_{Rk}$  is calculated from a fully yielded cross-section as in Eq. 7. This situation is reflected in Fig. 11a. Here,  $A_f$  and  $A_w$  are the beam flange and the beam web areas. Additionally,  $f_{yf}$  and  $f_{yw}$  are the yield strength in flanges and web.

$$M_{Rk} = f_{yf}A_f(h_w + t_f) + f_{yw}A_w h_w / 4 \tag{7}$$

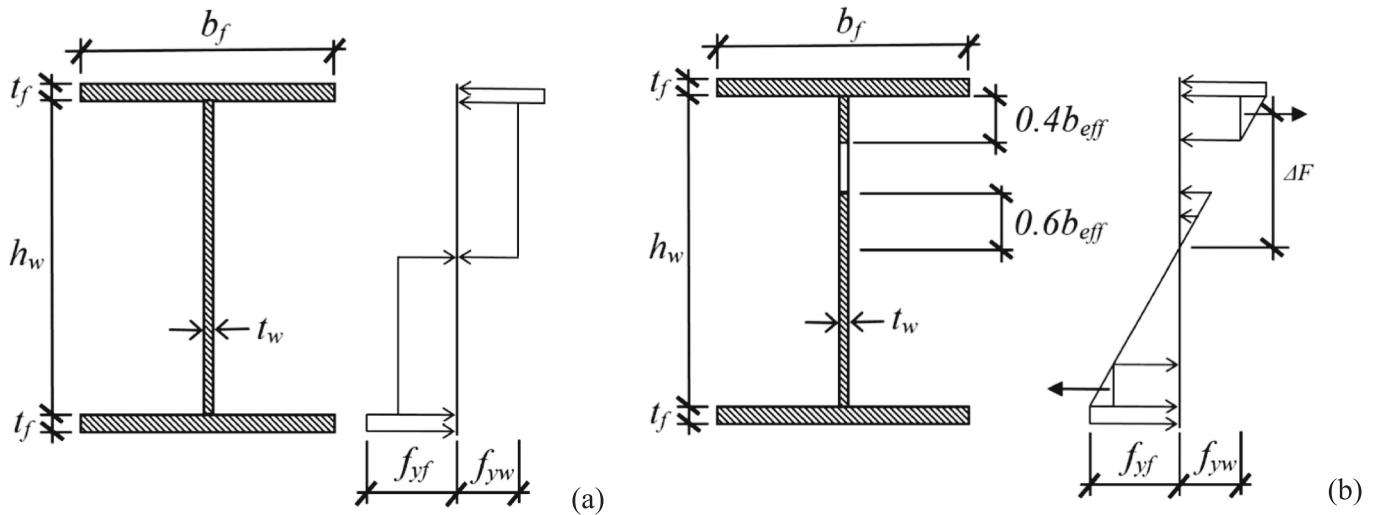


Fig. 11. Hybrid I-girder in cross-section (a) class 1 and (b) class 4. Here,  $b_{eff}$  is the effective plate width. Adapted from [109].

#### 4.1.2. Cross-section classes 3 and 4

In this case, the flanges are assumed to be class 3 or lower. As mentioned, the effective web cross-section must be calculated using the yield strength of the compression flange. In this situation, the resulting effective cross-section is usually not symmetric, and an iterative process would perform the strength calculation. However, approximate formulas for the flexural strength of an I-beam with equal flanges have been published in [139], which have been adjusted to the strength predicted by EC3-1-5 in [140] (see Eq. 8).

$$M_{Rk} = f_{yf} (w_{eff} - \Delta W) \quad (8)$$

Were:

$$w_{eff} = W \left[ 1 - 0.1 \frac{A_w}{A_f} \left( 1 - 124 \epsilon \frac{t_w}{h_w} \right) \right] \text{ when } h_w / t_w > 124 \epsilon \quad (9)$$

$$\Delta W = h_w A_w (1 - f_{yw} / f_{yf})^2 (2 + f_{yw} / f_{yf}) / 12 \quad (10)$$

$$\epsilon = \sqrt{235 / f_{yf}} \quad (11)$$

According to Eurocode 3-1-5, the elastic modulus of section  $W$  should be calculated in the midplane of the flanges. The formulas can be safe since they assume that the reduction in strength due to buckling and premature web deformation accumulates. As shown in Fig. 11b, this is only if the web yielded zone is  $< 0.4b_{eff}$ . Here,  $W_{eff}$  is the effective elastic modulus of the section.

#### 4.1.3. Lateral-torsional buckling

In this case, the reduction factor for lateral buckling is the same as for homogeneous beams [127]. It should be applied to the cross-section bending resistance calculated according to the abovementioned rules. The slenderness parameter can be calculated from Eq. 12.

$$\lambda_b = \sqrt{M_{Rk} / M_{cr}} \quad (12)$$

Were  $M_{cr}$  is the critical bending moment according to elastic stability theory calculated with gross cross-section properties.

**Shear resistance, patch loading and interaction shear-bending:** To calculate shear resistance and resistance to patch loading, Eurocode 3-1-5 provides formulas that already account for the flanges and web yield strengths. For the class 3 and 4 sections, the rules for the interaction between shear and bending can be applied without modification. However, for the class 1 and 2 sections, there is a debate regarding whether to neglect this interaction, despite suggestions to the contrary

in [138]. In practice, this interaction has been ignored in the USA and Sweden.

**Fatigue resistance:** The local yield strength that may occur in the web of hybrid beams will not affect their fatigue resistance [141,142]. It is because elastic behavior is observed after the first cycle. However, it should be noted that the stress range limitation of  $1.5f_y$ , as stated in Ref. [1], should only apply to the yield strength of the flange. Experimental evidence supporting this interpretation can be found in [141].

**Serviceability requirements:** Yielding of the web may occur during the serviceability limit state, reducing the girder's stiffness. However, since the stress level in this state does not exceed  $0.7f_{yf}$ , and the flange strength is limited to twice the web strength, the reduction in stiffness will be minimal, and deformation calculations need not consider yielding [143]. After a first loading cycle, the response to subsequent loading cycles not exceeding the first will be linear. It is explained by the residual stresses that accumulate during unloading. These residual and applied stresses will remain elastic unless the first load is exceeded. In [141], an experiment is performed where reversible behavior is demonstrated at a very high load level. In this case, the beam has a hybrid ratio of 1.67.

On the other hand, using HSS results in more significant deformations, which may impede the full utilization of their strength. However, one countermeasure that can be taken is to pre-camber the girder, mainly if deflection limitations are in place to improve appearance or provide drainage of water, for example. Alternatively, composite girders can be effective, as the increase in stiffness due to composite action is essential in reducing deformations. For more detailed information, refer to the code and the referenced publications.

#### 4.1.4. American approach

In the American approach, some standards still need to include how to consider hybrid sections explicitly. One of these standards is AISC 360-16, where, even though the yield strength between both flanges and the web are differentiated, there is no precise information on how to proceed with the effects of implementing a hybrid section. It is logical since this code is focused on buildings, where generally, the spans to be covered by the elements are not excessively large, and the use of these sections loses relevance. On the other hand, bridge construction does use beams that cover relevant spans. That is why a code that contemplates hybrid beam design is the AASHTO [144]. Among the recommendations stated here, it is said that "it is recommended that the difference in the specified minimum yield strengths of the web and the higher strength flange preferably be limited to one steel grade. Such sections generally are believed to have greater design efficiency". However, in the study

conducted by Barth et al. [32], before the publication of this standard, it was established that with a configuration using 690 W steel in the flanges and 480 W in the web, substantial weight and, therefore, material savings are achieved.

Gregor P. Wollmann [145,146] conducted studies that follow a similar approach to the one described in the section regarding European considerations. The recommended aspects outlined in AASHTO serve as the basis for these studies. According to [145], standard linear-elastic beam theory can be used to determine bending stresses in noncompact sections. However, the web capacity may be exhausted in some instances before the flanges reach their limiting stress. This phenomenon occurs with hybrid sections, where one or both flanges have a higher yield strength than the web, and sections with slender webs that may buckle laterally (bend buckling). In such cases, the flange stresses calculated using linear-elastic beam theory must be adjusted to account for the reduced contribution of the web to the overall section resistance. Once the web starts yielding, the flanges must resist a more significant portion of the bending moment. It can be viewed as a load transfer from the overstressed web to the adjacent flanges. While this condition is acceptable for most strength limit states, it renders linear-elastic beam theory inadequate. Stresses calculated using this theory must be adjusted for the load transfer effect. According to a simply supported beam model, the excess force carried by the web is distributed to the flanges with the supports of the beam located at the centroids of the flanges. An alternative approach is to assign the entire excess web force to the immediately adjacent flange, which provides a rough but simpler approximation. Since the excessive web force is usually insignificant, either approximation is acceptable.

AASHTO approaches this problem differently by defining a hybrid factor  $R_h$  as in Eq. 13.

$$R_h = \frac{\text{yield moment accounting for web yielding}}{\text{yield moment ignoring web yielding}} \quad (13)$$

AASHTO provides a set of complex equations for computing  $R_h$ . However, as the above approach is more straightforward and user-friendly, those equations are not presented here. Using the hybrid factor, the flange stresses in a hybrid section are given by Eq. 14. Here,  $f_b$  is the flange bending stress based on elastic beam theory.

$$f_b + \Delta f_{b,h} = \frac{f_b}{R_h} \quad (14)$$

In [146], an example of a design of a deck floor beam with a hybrid section is developed. Refer to the above reference to find out how the hybrid section phenomenon is included in the design, according to the above principle.

#### 4.2. Fabrication of hybrid girders

According to Veljkovic and Johansson [109], one of the problems that can arise from using girders with hybrid cross-sections is using HSS. Fabrication of elements made of these steels requires more stringent procedures than using low-strength steels. Although welding different steel grades is not a practical problem, it is essential to clarify the definition of matching electrodes. Standards related to steel elements describe guidelines for using matching electrodes based on the quality of steel used. It is recommended to use electrodes that match the web's strength to ensure the web-flange weld's strength. The general recommendation is to design the weld to fit the strength of the web steel grade.

On the other hand, although welded joints have the advantages of labor and material savings and the absence of drilling and overlapping, there are other aspects to consider. For example, weld properties, especially sectional residual stress, significantly influence the mechanical performance of structural steel members, such as structural stiffness, fatigue failure, and compressive member stability. Therefore, it is essential to determine the magnitude and distribution of sectional residual stress for HSS structures [147].

## 5. Discussion of the results

Hybrid beams in construction have seen significant research over the past two decades, with a heightened focus on Asia and Europe. In contrast, in America, it has lost the motion it had at the beginning. The surge in research in Asia, especially in recent years, is noteworthy. In the early years of research in the 1940s and 1960s, experimentation was the primary method of investigation, with some studies also incorporating computational analysis. During this period, regulations were also revised and updated to provide guidelines for further research. With the advancement of computing technology, computational research has become increasingly crucial in the past decade, allowing for the simulation of behavior and the generation of conclusions at a lower cost. While experimental research has lost some of its prominences, it remains essential in verifying results and ensuring accuracy. Computers provide a valuable tool for simulation, but experimental verification in a laboratory setting is crucial.

Regarding the various aspects explored in this research, examining internal forces in beams (bending, shear, and their interplay) is a prominent topic. The research aims to investigate the behavior of the beams, stress redistribution, and possible post-critical behaviors. The findings help establish a solid theoretical framework to support the design of hybrid beams and their compliance with standards. Of the studies conducted, 69% focus on internal stress in beams, with a particular emphasis on bending, constituting 36% of the investigations.

Among the other topics included in this research are those related to economic and optimization concepts, with incidences of 4% and 8%, respectively. All studies highlight the economic advantages of using beams with different yield strengths in flanges and webs. It is important to note that there are still gaps in the research related to this topic. Considering the wide range of real applications of hybrid beams, this type of study should be further deepened, mainly focused on real applications. Alternatively, some studies focus on fatigue and the use of high-strength steels in specific applications, such as composite beams and behavior in fire. Another group of studies examines local buckling phenomena. The limited number of studies on compression is because the radius of gyration of the section, the unbraced length, or the overall yield strength of the section primarily determines compression resistance. As a result, hybrid columns are not efficient in purely compression conditions. However, as the amount of bending supported by the column increases, the more beneficial the use of hybrid columns becomes.

The literature review found a wide range of yield strengths for both the web and flanges. Most of the studies focused on values between 200 MPa and 500 MPa, with a higher concentration of web yield strengths in the range of 200 to 300 MPa. The range of flange yield strengths is higher, reaching between 400 and 500 MPa, which is in line with typical hybrid beam configurations. Hybrid ratios in the review vary from below 1 to nearly 8, with most of the studies falling between 1 and 2. The most significant concentration of studies is in the range of 1.3 to 1.6, where it has been discovered that hybrid beams perform best. When the hybrid ratio is lower than 1, the web of the beam has a higher yield strength than the flanges, which can lead to inefficiency in bending as the flanges may reach their limit without the web being fully utilized. As the hybrid ratio increases, the panel length between stiffeners must decrease to anchor the stress field and achieve post-critical resistance states that maximize the web's capacity. However, this increase in the number of stiffeners for stiffness values greater than two starts to become uneconomical. For stiffness values  $>3$ , the possibility of fully exploiting the capacity of the flanges is reduced due to local web failure. Consequently, using very stiff beams is neither economically nor structurally efficient.

Even when the different categorical variables and some relationships have been analyzed separately, handling so much information is difficult. For this reason, statistical analysis is used to summarize all this information and express it in deeper relationships (difficult to see with the naked eye) between variables. Correspondence analysis, a

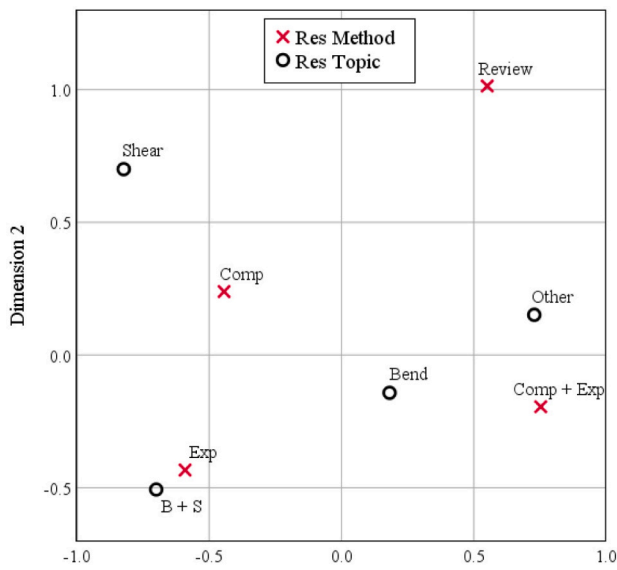
descriptive statistical method, is used to visualize contingency tables in low-dimensional spaces. This method allows the creation of a perceptual graph that demonstrates the relationship of dependence or similarity between variables based on the distance between points.

A simple correspondence analysis (SCA) is developed in this case, where only two categorical variables are related. Three simple rules are used to interpret such graphs: (a) the closer the value of the variable is to the origin, the more common its use is, (b) the relationship between the values of two variables is more significant as each point approaches another point, and (c) the relationship is more exclusive the farther the points are from the origin. Two measures are used to ensure that the experiment is statistically valid. The significance level of the test should be  $<0.05$ , which indicates that there is statistical significance. On the other hand, the cumulative proportion of inertia allows us to check if there is a good representation of the data in the two analyzed

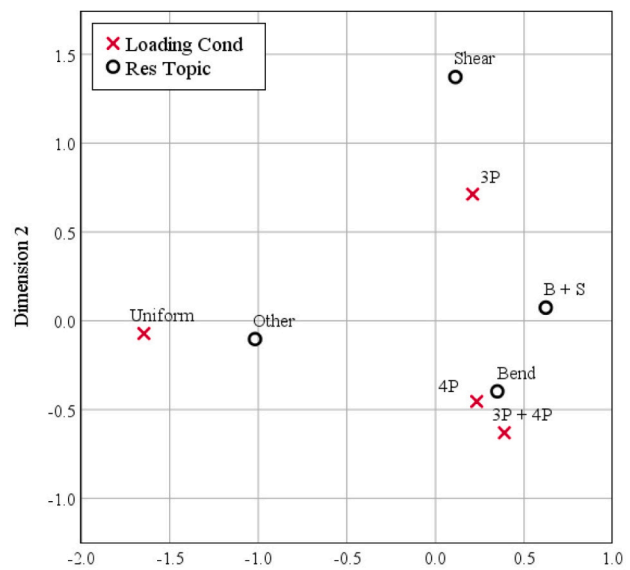
dimensions.

Fig. 12 shows three SCAs between different categorical variables based on the research topic. In this way, all the categorical variables are related through a single one. There is the possibility of performing multiple correspondence analyses, but having a sufficiently large database is usually necessary. The graph shown in Fig. 12d is a summary of all these relationships.

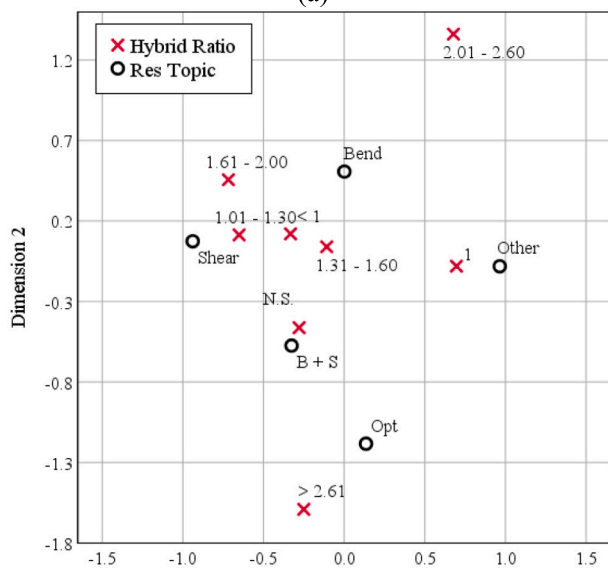
Fig. 12a depicts the relationship between the research topic and the method used for its development. Here it is shown that all methodologies influence the study of bending. It is because of the proximity of the point to the origin. In the summary graph of the figure, this is represented by a golden arrow. The study of pure shear was mainly conducted through computational methods, while the interaction between bending and shear was primarily explored through laboratory experiments. On the other hand, broader studies, such as those on buckling and fatigue,



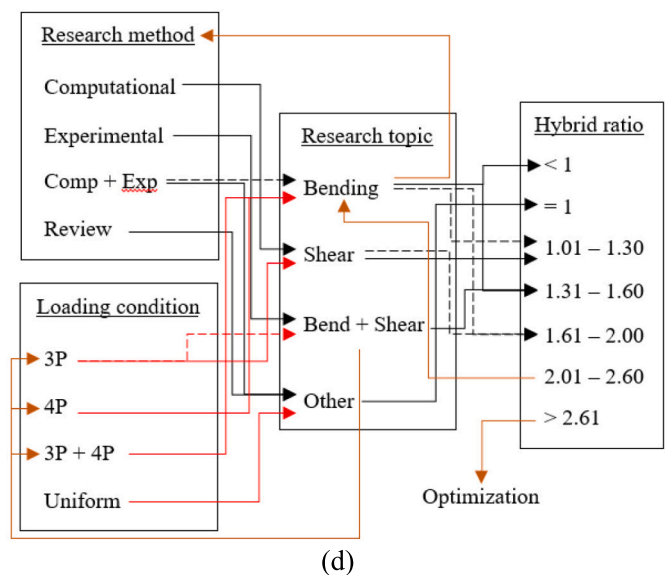
(a)



(b)



(c)



(d)

Fig. 12. Simple correspondence analysis: (a) research method, (b) loading conditions, and (c) hybrid ratio in comparison with the several research topics; (d) is a graphical summary of the results obtained in SCA. Here, solid and dashed lines denote strong and weak relationships, respectively. Golden arrows show special relationships.

were mainly carried out using computational methods and were validated through practical applications. Additionally, these studies have a relationship with review-based investigations.

On the other hand, the relationship between the loading conditions and the research topic is evident. Fig. 12b shows that bending has been studied using 4P loading configurations and combining it with the 3P type. The shear is associated with 3P arrangements, while the interaction has no specific configuration (perhaps 3P, based on its position concerning the origin). The equidistance with the three points confirms that the three types of experimentation are indistinctly associated with the interaction. It is represented in the graphical summary by a golden arrow. Finally, other studies are related to uniform loading, especially those focused on more general phenomena such as optimization.

Finally, each research topic has a corresponding level of hybridization. Bending has been associated with several ranges of hybrid ratios, mainly below 1 and between 1.31 and 1.60. Several studies have referred to the latter as the optimal range for bending. Here, post-critical behaviors are balanced, and there is no premature failure in any components. In the case of shear-related studies, there is a strong association with hybrid ratio intervals 1.01–1.30 and 1.61–2.00. These intervals are where the web fails in shear before fully utilizing its capacity. Hence, these intervals are studied primarily to comprehend the sections' failure modes and post-critical behavior. Regarding the interaction of internal forces, the relationship with the different hybridization values is less intense than in the cases of pure bending and shear. Still, equal closeness is denoted with all of them. It shows that for the interaction, the studies about hybridization are balanced for hybrid ratio values  $<2$ . In other studies, a close relationship with homogeneous elements is evident. It is crucial first to study certain phenomena or applications in their simplest form and then apply the findings to the more complex domain of hybrid structures. It is worth mentioning that the value of  $R_h = 1$  serves as a control, as results from hybrid beams are often compared to those of homogeneous ones. As a result, this value relates to all three types of stresses: bending, shear, and interaction. The hybridization ratios between 2.01 and 2.60 are mainly related to bending studies and, to a lesser extent, to other types. It is also noteworthy that elements with a hybrid ratio  $>2.60$  are mainly associated with results from optimization processes.

This analysis provides a broad understanding of the research on hybrid steel beams. By utilizing the different correspondence analyses and the summary diagram, we can arrive at general conclusions regarding the progression of research in this area. Considering a specific research topic, a particular type of research is designed with specific loading conditions, resulting in the implementation of steel beams with specific hybrid ratios.

Regarding the practical considerations in the design and construction of these elements, it has been observed that important standards still need to contemplate how to achieve hybrid configurations explicitly. On the other hand, the standards that deal directly with the subject only include some of the information required for a complete design. It is necessary to gather information in separate investigations. It limits the practical applications of these configurations. On the other hand, it has been established that mixing various types of steel is not directly a construction problem. The problem lies in the fact that these configurations generally include HSS, which is an aspect that could complicate the procedure.

Finally, several research gaps can be identified with all the information gathered. First, more laboratory research is needed to generalize results on steel-steel hybrid configurations in simple girders. These results should be reflected in the different standards related to steel construction. From here, other more complex structures (e.g., reinforced steel girders or complex box girders) can benefit from the advantages of hybrid configurations. On the other hand, there are indications of the most recommendable hybrid ratios. However, without research on obtaining optimal design results for these elements, there are still doubts about implementing a rational hybrid configuration. Therefore, the two

main research lines in this field focus on updating the different steel codes. Then, performing complex optimization studies (using environmental criteria, LCA) to deepen the best practices when implementing a hybrid configuration. Consequently, incorporating these "basic" elements in other more complex typologies will significantly help improve building sector sustainability indexes.

## 6. Conclusions and promising lines of research

The ability of hybrid steel girders to fully utilize their capacity to cope with shear and bending stresses is a great advantage when designing these elements. Research in this field and its application in the design of structures has been a growing topic over time. Several researchers have been interested in this beam type, seeking to generate more efficient design methods that comply with different countries' technical specifications and regulations. However, although their use guarantees numerous advantages, some research gaps must be addressed, e.g., their consideration in the different standards or their application in actual structures (e.g., complex girder structures).

Research development has prioritized understanding the internal behavior of hybrid elements under shear and bending stresses. The studies conducted have allowed the generation of a theoretical framework that supports the use of hybrid girders and provides a theoretical understanding of the strength of the elements under different loading conditions. In contrast, research on these elements' economic (or environmental) benefits and design optimization processes has yet to be widely published.

In addition to presenting the bibliometric results, this review analyzes categorical variables related to high-strength steel beams, especially those with hybrid configurations. The analysis of these variables allows putting into context the main *research topics*, the *methods used for their development*, the main *loading conditions* used in the experiments, the distribution of the different *kinds of steel in both the flanges and the web*, and the corresponding *hybrid ratio*. As a main result, it is summarized that for  $R_h$  values less than or equal to 1 in elements working in bending, the flanges will be exhausted when the web has not developed its maximum capacity. For elements with  $R_h$  values  $<1.3$ , these overstress mechanisms will not develop. The element's behavior will be similar to a homogeneous one due to the slight difference between the elastic limits. For  $R_h$  values  $>2$ , developing maximum capacity in the flanges is challenging before the web is exhausted. Several authors have established that the interval  $1.3 < R_h < 1.6$  is where the best performance of hybrid steel girders is obtained.

On the other hand, simple correspondence analysis is implemented to find underlying relationships between the categorical variables. This way, links have been established that help to generalize the study. Based on the interaction of the data extracted from the 128 papers analyzed, these results serve as a practical guide for future research.

Finally, the main line of research should focus on validating results in designing girders with hybrid configurations. These results should be introduced in the codes related to the construction of steel structures. Additionally, given the scarcity of research analyzing the economic or environmental benefits, the possibilities of applying design optimization procedures, sustainability studies, or the LCA of structures composed of hybrid elements, this subject has an immense field of application to improve sustainability indexes in the building sector. It is also attractive to develop research to verify and propose heuristic and meta-heuristic optimization strategies applied to design these elements, emphasizing the formulation of variables that develop the concept of hybridization. It is also possible to extend the research concerning hybrid box girders. Another field that can be explored in hybrid elements corresponds to the approach of longitudinal hybridization instead of the transverse one developed throughout this work.

## CRedit authorship contribution statement

**Agusztine Terreros-Bedoya:** Conceptualization, Methodology, Investigation, Formal analysis. **Iván Negrin:** Conceptualization, Methodology, Investigation, Formal analysis, Visualization, Writing – original draft. **Ignacio Payá-Zaforteza:** Writing – review & editing, Supervision, Project administration. **Víctor Yepes:** Supervision, Project administration, Writing – review & editing, Funding acquisition.

## Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## Data availability

Data will be made available on request.

## Acknowledgments

This work was supported by the grant PID2020-117056RB-I00, which was funded by MCIN/AEI/10.13039/501100011033 and by “ERDF A way of making Europe”. Grant PRE2021-097197 funded by MCIN/AEI/10.13039/501100011033 and by FSE+.

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