

RESEARCH ARTICLE

Parametric study of concrete box-girder footbridges

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Abstract

This paper presents a study of the parametric variability of post-tensioned concrete box-girder pedestrian bridges. SAMO2 algorithm is used for the parametric study. This algorithm combines SA with a mutation operator, to find the economic solutions. A span-length parametric study analyzes the characteristics for the best design of a three-span deck in which the main span ranges from 30 to 60 m. The depth and the number of strands were adjusted according to span length, while the thickness of the slabs presented the same optimum values in all cases. Results show that the amount of steel and volume of concrete per square meter of deck shows a good correlation with the main span length. This study demonstrates that by increasing the main span length by one meter, the total cost per square meter of the deck increases by 6.38 euros on average. Thus, this paper shows the relationship between the span length and geometrical and steel variables to produce and build a cost-efficient pedestrian bridge.

Keywords

Structural optimization; Post-tensioned concrete; Box-girder bridge; Pedestrian bridge

Received: 15 May 2018; Accepted: 22 June 2018

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1. Introduction

Box girder bridges are commonly used bridge deck systems because of cost-effective and aesthetic solutions. The first continuous prestressed box-girder bridge was the Sclayn Bridge over the river Maas, built by Magnel in 1948 with two spans of 62.7 m. From that date, the box girder has been found in beam, portal frame, arc, cable-stayed and suspensions bridges. These decks can be constructed using many methods. Concrete box girders are usually cast in situ, or precast in segments and then erected and prestressed [1]. Stationary falsework and traditional scaffolding is frequently used when the superstructure is not at a

great height above the ground or the number of spans is limited [2].

The applicability of this type of bridge has grown steadily due to its strength against positive and negative bending moments and to torsional stresses. These structural characteristics combined with a low dead load have led this type of bridge to be one of the ones with the most widespread use nowadays. Indeed, the use of continuous concrete box-girder bridges has increased recently [3]. Thus, much research has been devoted to promote understanding and achieve a proper design. During the 1980s and 1990s, researchers focused on the structural analysis [4-7], since the box girder presented a complex problem. Later, other research

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topics arose, such as, the structural behavior at different construction stages [3,8], the durability conditions and their effect on the strength conditions [9,10], deflections during construction [11], long-term behavior [12], life-cycle assessment [13,14] and multi-criteria decision making methods applied to the sustainable bridge design [15].

Some authors have published recommendations for the geometry and material of prestressed concrete bridge design [2,16-18]. These guides are useful in the first stage in the design process. Once the geometrical layout is a priori defined, the stresses and displacement are compared with the allowable values, according to the code specifications [19,20]. Should the initial design dimensions or material grades be insufficient or excessive, the structure is redefined on a trial-and-error basis. The final design does not guarantee to be optimal. For that reason, structural optimization methods are clear alternatives to designs based on experience. However, finding the optimal design is a difficult and challenging task since it requires deciding on many variables such that the design goals are fulfilled while many constraints are satisfied. In fact, these methods are seldom adopted by practicing engineers, who prefer intuition-based trial-and-error methods to optimization algorithms. A review of heuristic algorithms applied to structural optimization is presented in the study of Hare et al. [21]. For the complex and realistic non-linear structural optimization problems that are investigated in this study, these algorithms appear to be the only reliable approach.

Srinivas and Ramanjaneyulu [22] minimized the cost of a T girder bridge deck using artificial neural networks and genetic algorithms. Hernández et al. [23] optimized the design of prestressed concrete beams. Martínez-Martín et al. [24] proposed a variant of ant colony optimization to optimize tall bridge piers. Martí-Vargas et al. [25] predicted the transfer length of prestressing strands with neural networks. Rana et al. [26] presented an evolutionary operation, based on a global optimization algorithm for the minimum cost design of a two-span, continuous, prestressed concrete I-girder bridge structure. A hybrid

simulated annealing algorithm [27] and a memetic algorithm with variable-depth neighborhood search [28] were applied to find the most economical solution for precast-prestressed concrete U-beam road bridges. A glowworm swarm algorithm was used to optimize concrete I-beams [29]. However, studies on the optimum box girder design are comparatively scarce [30-33]. Hassanain and Loov [34] present a review of cost optimization of concrete bridge infrastructure.

In addition to the above, the use of other materials, like high-strength concrete, can alter the design criteria [35,36]. The use of high-strength concrete reduces self-weight and, therefore, the amount of concrete and steel required. To this end, this study employs the SAMO2 heuristic optimization technique to find the best economic designs for a box-girder pedestrian bridge made with different concrete grades for a three-span deck in which the main span ranges from 30 to 60 m. For this purpose, a program encompasses modules for structural analysis, limit states verification and cost evaluation. We demonstrate not only the applicability of the algorithm to the structure, but also we provide practicing engineers with guidelines for the efficient design.

2. Optimum bridge design problem

This study minimizes the cost of a continuous beam deck with a box-girder cross section of post-tensioned concrete. A candidate design is defined by design variables (x_1, x_2, \dots, x_n). The problem involves an objective function F of Eq. (1), subject to the constraints represented by Eq. (2).

$$F(x_1, x_2, \dots, x_n) \quad (1)$$

$$G_j(x_1, x_2, \dots, x_n) \leq 0 \quad (2)$$

The objective function evaluates the cost for the total number of construction units (r), considering material and placement costs listed in Eq. (3).

$$C = \sum_{i=1,r} p_i \times m_i(x_1, x_2, \dots, x_n) \quad (3)$$

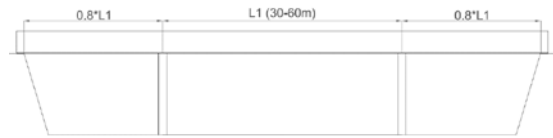
Unit prices (p_i), shown in Table 1, were obtained from the BEDEC ITEC database (Catalonia Institute of Construction Technology,

Table 1. Basic prices

Unit measurements	Cost (€)
m ³ of scaffolding	10.02
m ² of formwork	33.81
m ³ of lighting	104.57
kg of steel (B-500-S)	1.16
kg of post-tensioned steel (Y1860-S7)	3.40
m ³ of concrete HP-35	104.57
m ³ of concrete HP-40	109.33
m ³ of concrete HP-45	114.10
m ³ of concrete HP-50	118.87
m ³ of concrete HP-55	123.64

2015). Concrete unit prices were determined for each compressive strength grade according to the mix design, including the cost of raw materials extraction, manufacture and transportation. The measurements (m_i) concerning the construction units depend on the design variables.

This study describes the cost minimization of a pedestrian deck bridge with three spans and a low height. The three spans into which the deck length is split follow the relation that the external span is 80% of the central (Fig. 1). The pedestrian bridge is 3.5 m wide. This problem includes 33 discrete design variables that define the geometry, the concrete, the reinforcing steel bars and the post-tensioned steel. The parameters define the overall geometry, the materials, the loads acting on the structure, the exposure class, and the Spanish codes for structural concrete and bridge design loads followed by this study [19,20]. To this end, a program analyzes and evaluates the serviceability and ultimate limit states (SLS and ULS) and the geometrical and constructability requirements. This modulus verifies the demands of the safety, as well as those relating to the aptitude for service requirement. The structure was modelled by a linear element with 30 bars, 31 sections and 93 degrees of freedom. Transverse analysis was carried out using a frame model. Besides, the minimum amount of reinforcement for the stress requirements and the geometrical conditions are also examined. The readers are encouraged to refer to our previous work [30] for detailed description of the design

**Fig. 1.** Three span post-tensioned concrete box-girder bridge

variables, the parameters, and the structural analysis and constraints.

3. SAMO2 optimization algorithm

The heuristic method used for this study is a hybrid SA algorithm with a mutation operator (SAMO). SAMO, proposed by Martí et al. [27], takes both of the benefits of good convergence in simulated annealing and enhancing diversity of genetic strategy. SAMO takes both of the advantages of good convergence in simulated annealing and enhancing diversity of genetic strategy. Here, we propose an extended version of SAMO, called SAMO2. SAMO2 accepts worse solutions when the increment is lower than a value, which depends on the temperature and a stochastic variable. Therefore, this probabilistic jumping property facilitates global optimum searching. Eq. (4) gives the transition rule of this method.

$$p_{SAMO2} = \frac{1}{1 + e^{\frac{\Delta E}{T}}} \quad (4)$$

The calibrated method involves random variation of up to 50% of the variables, the initial temperature by Medina's method, Markov chains of 5000 iterations, a cooling coefficient of 0.80 and a stop criterion of three Markov chains without improvement. The algorithm was coded in MATLAB with an INTEL® Core™ i7-3820 CPU processor and 3.6 GHz. Computer runs were performed nine times, according to the extreme value theory methodology used by Carbonell et al. [37], to obtain minimum, mean and standard deviations of the random results. The difference between the minimum cost obtained after nine runs and the extreme value estimated was €509.53, a difference of just 0.34% compared to the theoretical minimum value. From the point of view of the

structural engineer, this indicates that the difference is small enough to make the solution provided by the proposed SAMO2 acceptable.

4. Best results for the span-length parametric study

This section presents the best solutions for five three-span bridges with the main span ranging from 30 to 60 m. Table 2 gives the best geometrical characteristics obtained after nine computer runs for each case. Table 3 summarizes the mean values to detail the information. The depth and the number of strands depended on the span length. The mean values for the depth varied between 1/34 and 1/39 in relation to the main span and the ratio N_s/L_1 ranged from 0.93 to 1.70.

The width of the bottom slab tended to the minimum (1.4 m) to minimize the weight. However, the geometrical restrictions conditioned by the thickness of the webs and the haunches forced this width to be increased. Regarding the width of web inclination (d), there is no good correlation between this variable and the minimization of the cost for each span length. The vertical web reduces the amount of concrete and, therefore, the total cost. Nevertheless, the web inclination reduces the cantilever length and the width of the bottom slab. Consequently, the longitudinal shear is decreased and there are less

depth requirements. Therefore, this variable can be adjusted for esthetic reasons without major economic consequences.

The thickness of every slab, except the webs, tended to adopt the minimum value proposed by the optimization program (150 mm). It is only advisable to increase the thickness for geometrical reasons, for instance, when the distance from the reinforcing bars is a determining factor. The web thickness took values between 400 and 460 mm. This value enables strands to fit in this thickness and guarantee the structural and constructability requirements. Comparing these results with some recommendations, it is worth noting that Ministerio de Fomento [17] suggested slab thickness greater than 200 mm for road bridges and web thickness of more than 300 mm or 6 cm/m total width. Schlaich and Scheff [2] proposed deck and bottom slabs greater than 200 mm and 150 mm, respectively. Regarding webs, the thickness suggested should not be less than 300 or $200+2*\varnothing_d$, where \varnothing_d is the duct diameter. Note that loads for pedestrian bridges are smaller and therefore, the thickness was limited in this study to 150 mm.

The concrete type for the best solutions varied between 35 and 55 MPa, independent of the span length. However, analyzing the mean values, a concrete strength increment with the span length was highlighted.

Table 2. Best geometrical variables for 30-60 m span

L_1 (m)	L_2 (m)	h (m)	b (m)	d (m)	e_s (m)	e_v (m)	e_i (m)	e_a (m)	t (m)	f_{ck} (MPa)	L_{PI}	N_s
60	48	1.65	1.50	0.00	0.15	0.15	0.15	0.45	0.30	45	0.05	100
55	44	1.50	1.60	0.00	0.15	0.15	0.18	0.43	0.28	50	0.10	76
50	40	1.25	1.45	0.28	0.15	0.15	0.15	0.43	0.28	55	0.15	68
40	32	1.10	1.45	0.00	0.15	0.15	0.15	0.45	0.15	40	0.20	44
30	24	0.90	1.40	0.00	0.15	0.15	0.18	0.40	0.18	35	0.20	26

Table 3. Mean geometrical variables for 30-60 m span

L_1 (m)	L_2 (m)	h (m)	b (m)	d (m)	e_s (m)	e_v (m)	e_i (m)	e_a (m)	t (m)	f_{ck} (MPa)	L_{PI}	N_s
60	48	1.68	1.55	0.04	0.16	0.16	0.19	0.46	0.29	54	0.09	102
55	44	1.42	1.52	0.11	0.15	0.15	0.16	0.45	0.30	54	0.12	83
50	40	1.28	1.43	0.21	0.15	0.15	0.16	0.43	0.28	52	0.13	70
40	32	1.10	1.48	0.03	0.15	0.15	0.16	0.45	0.16	42	0.20	46
30	24	0.89	1.54	0.24	0.15	0.15	0.16	0.40	0.16	38	0.19	28

The number and dimensions of the ducts conditioned the haunch. The 30-m case and 40-m case had one duct per web. Therefore, the mean limitation was the thickness of the bottom slab. However, other cases presented two ducts per web; this led to an increase in the haunch to provide space for ducts on the high and low points.

With regard to the point of inflection LPI, this was placed at 0.2 times the span length from the pier for the 30-m case and 40-m case. Nevertheless, as the span length increased, this distance was reduced. The results suggest that for longer spans the inflection point is nearer to the pier to intensify the prestressing steel effect in the piers.

Tables 4 and 5 summarize the best and mean measurements to be taken into account for the footbridge design. The mean measurements presented better correlation with the span length than the best measurements. Increasing the main span length by one meter resulted in an increment of 6.38 euros per square meter of the deck. Fig. 2 shows the relationship between the cost per square meter of deck (C) and the main span length ($C=6.3808L_1+33.296$ with a regression coefficient $R^2=0.9258$).

The amount of concrete and the post-tensioned steel had a direct relation to the span length, since these measurements depended on the depth and the number of strands, and these variables in turn, were

intimately related to the span length. Fig. 3 shows the correlation between the amount of steel and the volume of concrete per square meter of the deck and the main span length.

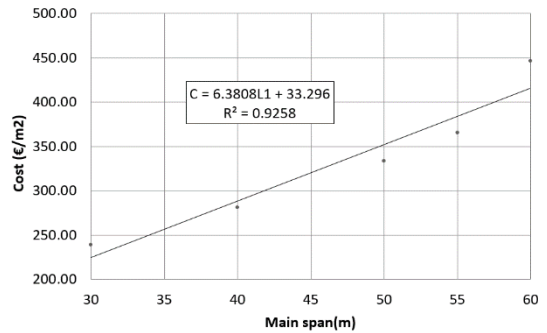


Fig. 2. Mean cost per m² of deck according to the main span length

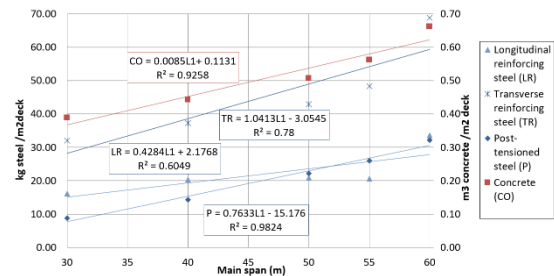


Fig. 3. Mean amount of steel and volume of concrete per m² of deck according to the main span length

Table 4. Best measurements for the 30-60 m span solutions

L_1 (m)	Cost (m)	Cost (€/m ²)	Concrete (m ³ /m ²)	Post-tensioned steel (kg/m ²)	Longitudinal reinforcement (kg/m ²)	Transverse reinforcement (kg/m ²)
60	215393.49	391.48	0.62	31.49	23.72	45.15
55	179063.66	354.79	0.57	23.93	18.49	43.83
50	151043.99	328.93	0.50	21.41	18.38	42.73
40	100368.53	272.59	0.44	13.85	18.26	34.88
30	65230.57	235.32	0.39	8.19	19.16	29.49

Table 5. Mean measurements for the 30-60 m span solutions

L_1 (m)	Cost (m)	Cost (€/m ²)	Concrete (m ³ /m ²)	Post-tensioned steel (kg/m ²)	Longitudinal reinforcement (kg/m ²)	Transverse reinforcement (kg/m ²)
60	245763.61	446.68	0.66	32.19	33.65	68.84
55	184496.87	365.56	0.56	26.03	20.58	48.29
50	153146.55	333.51	0.51	22.11	20.88	42.91
40	103560.62	281.26	0.44	14.34	20.37	37.34
30	66237.03	238.95	0.39	8.82	16.10	32.04

The amount of post-tensioned steel per square meter of the deck is represented by a good linear function ($P=0.7633L_1-15.176$ with a regression coefficient $R^2=0.9824$). The volume of concrete per square meter of deck also has a good correlation with the span length ($CO=0.0085L_1+0.1131$ with a regression coefficient $R^2=0.9258$). One meter of main span length involved a further 0.0085 m^3 of concrete per square meter of the deck and 0.76 kg more of post-tensioned steel per square meter of the deck. Longitudinal reinforcement presented less depending on span length, since post-tensioned steel was the biggest factor contributing to longitudinal strength. Therefore, the proportion of the variance of the reinforcing steel that is predictable from the deck length is lower. A linear relation may be used to describe the general trend for the longitudinal reinforcing steel per square meter of the deck ($LR=0.4284L_1+2.1768$ with a regression coefficient $R^2=0.6049$) and the transverse reinforcing steel per square meter of the deck ($TR=1.0413L_1-3.0545$ with a regression coefficient $R^2=0.78$). Comparing the mean and the best measurements (Tables 4-5), it is worth noting that the best values had, on average, 2% more longitudinal reinforcement, 4% less post-tensioned steel and 8% less transverse reinforcement.

5. Conclusion

This study has evaluated the optimum design of a three-span deck post-tensioned concrete box-girder pedestrian deck with a main span ranging from 30 to 60 m. The SAMO2 algorithm, which combines SA with a mutation operator, found the economic solutions. The findings in this study addresses that the depth and the number of strands for the optimum design depend on the span length, with a relation that varies between $1/34$ and $1/39$ for the depth and between 0.93 and 1.70 for the strands. Conversely, the slab thickness is not related to the span length. The web thickness ranges between 400 mm and 460 mm and the slab thickness is about 150 mm. The amount of steel and volume of concrete per square meter of deck shows a good correlation with the main span length. By increasing the main span length by one meter, the total cost per square

meter of the deck increases by 6.38 euros on average. Considering that the goal is addressing practitioners' practical needs, this parametric study provides design criteria to guide designers as to the characteristics of optimum box-girder pedestrian deck bridges. Besides, the algorithm is quite flexible to further structural designs instead of traditional try-and-error methods.

Acknowledgments

This research was financially supported by the Spanish Ministry of Economy and Competitiveness along with FEDER funding (Project BIA2014-56574-R and Project BIA2017-85098-R).

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