

# Heuristic design of a precast-prestressed concrete U-beam and post-tensioned cast-in-place concrete slab road bridges

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## Abstract

This paper proposes simulated annealing and threshold accepting procedures for the automatic design of two different bridge types. Both cases are prestressed concrete road bridge decks typically used in public road construction. Simulated annealing is first applied to a precast beam of 30–30 meters of longitudinal spans and 12.00 m of width. The beam has a double U-shape cross-section and a beam spacing of 6 m. This problem involves 59 discrete design variables for the geometry of the beam and the slab, concrete grade, reinforcing steel and prestressing steel. The simulated annealing method indicates savings of about 5% with respect to a traditional design. The second bridge case is a 20–36–20 m post-tensioned cast-in-place concrete slab road bridge deck. This example needs 33 discrete variables to define the complete structure. The threshold accepting method is used for the optimization. Our findings indicate savings of about 7.5% with respect to the design based on experience. Finally, the results show that heuristic optimization provides other options to reduce the design costs of real prestressed bridge decks.

*Keywords: precast-prestressed concrete, post-tensioned cast-in-place, U-shape cross-section, slab deck, heuristic optimization.*

## 1 Introduction

Since its conception in the mid-1950s, artificial intelligence has been an area of knowledge, which has developed into a wide range of fields such as project planning, troubleshooting conditional optimization, operation research, operation planning, logistics and transport networks, among others. Structural design is a



field with high potential, where artificial intelligence has already been successfully applied. Designing can be understood as a sequence of decisions that lead to the best selection of variables capable of meeting the strength requirements and functional demands to which this structure is subjected, while at the same time optimizing a set of design criteria.

Many traditional processes for structural concrete projects select initial solutions founded on material grades, cross-section magnitudes, and steel reinforcement based on formal common practice. Once the structure is well-defined, it follows the analysis of the structure and testing the passive and active reinforcement. Should the measurements, reinforcement or material grades be insufficient, the structure is redefined on a trial-and-error basis. This procedure is not automatic and leads to safe designs, but the cost of the concrete structures is, consequently, highly dependent upon the knowledge of the structural designer. Optimization methods are a clear alternative to experience-based techniques. However, it is worth noting that experience is critical for the progress of computer design models, since design comprises more than the mere use of codes of practice. This means that experience will change beyond preliminary design decisions to the judgment required to improve computer design models.

In this context, optimization techniques in the design of concrete structures lead to efficient designs, thus making them a very interesting application. These techniques can be classified as exact or approximate. The former is mainly based on mathematical programming and aims to reach the global optimum of a conditioned problem [1, 2]. These methods are very efficient when using few design variables, but the computational time for the calculation quickly reaches an unreasonable length when large variable numbers are at hand, as is the case for most real structures. Sarma and Adeli [3] provide an extensive review of articles related to optimization of concrete structures. The second group of techniques includes heuristics, whose recent development is linked to the development of artificial intelligence procedures. These methods include a wide variety of search algorithms, such as genetic algorithms [4], simulated annealing [5], ant colony [6], cloud particles evolution [7], and others. Heuristic techniques have been successfully applied in various areas of structural engineering; for example hydraulics, project planning and transport [8].

An extensive review of different structural optimization methods can be found in Cohn and Dinovitzer [9]. With regard to reinforced concrete structures, the first heuristic applications were focused on optimizing both supported reinforced concrete beams [10] and the study of three-dimensional reinforced concrete porches [11]. Recently, our research group has used heuristic algorithms in optimizing walls, arches, building frames, bridge piers and bridge decks of prestressed concrete roads in situ and precast [12–20].

This paper presents a case study focusing on the optimization of bridges. For this purpose, the two algorithms used are described, as well as the two practical examples. The result of this analysis is directly applicable to both similar cases by professional practices.



## 2 Heuristic algorithms for optimization

This heuristic algorithm finds the value of the variables that minimizes the cost and satisfies the following restrictions:

$$F(x_1, x_2, \dots, x_n) = \sum_{i=1, r} p_i * m_i(x_1, x_2, \dots, x_n) \quad (1)$$

$$g_j(x_1, x_2, \dots, x_n) \leq 0 \quad (2)$$

$$x_i \in (d_{i1}, d_{i2}, \dots, d_{iq_i}) \quad (3)$$

where  $x_1, x_2, \dots, x_n$  are the variables to be optimized (i.e. the design variables). Each design variable may assume the discrete values listed in Eq. (3). The objective function  $F$  defined in Eq. (1) is cost,  $p_i$  are the unit prices,  $m_i$  are the measurements,  $r$  is total of the construction units. The constraints  $g_j$  in Eq. (2) are all the service limit states (SLSs) and ultimate limit states (ULSs) with which the structure must comply, as well as the geometric and constructability requirements of the problem.

### 2.1 Simulated annealing (SA)

Simulated annealing (SA) was originally described by Kirkpatrick *et al.* [21]. The term “annealing” refers to the controlled process of heating and cooling a material. If there is a slight decrease in temperature, the metal acquires a crystal structure that corresponds to a thermodynamic minimum energy state. If it is cooled too fast, the molecules can reach a meta-stable state far from the appropriate settings. This analogy enabled the design of a heuristic optimization algorithm, considering that each solution reached the best state of energy or objective function. The acceptance criterion for new solutions is controlled by the Metropoli expression  $\exp(-\Delta E/T)$ , where  $\Delta E$  is the cost increase and  $T$  is temperature.

The algorithm starts with a randomly generated feasible solution and a high initial temperature. The initial working solution is modified by a small random movement of the value of the variables. The new solution is checked in terms of cost, accepting some higher cost when a random number between 0–1 is smaller than the expression  $\exp(-\Delta E/T)$ . If this solution is feasible against structural restrictions, it will be adopted as a new solution. The initial temperature is geometrically reduced ( $T=kT$ ) by a cooling coefficient  $k$ . At each temperature step, a certain number of iterations called a Markov chain is executed. The algorithm stops when the temperature is reduced to a small percentage of the initial temperature, and simultaneously, no improvement for a number of consecutive Markov chains is achieved (typically 1% and 1 or 2 Markov chains).

This method is able to overcome local optima at medium/high temperatures. It also gradually converges to the optimum when the temperature is reduced to zero. The SA method requires the calibration of the initial temperature, the Markov chain and the cooling coefficient. The parameters governing the heuristics are



described in Section 3. The initial temperature is adjusted using a method similar to that proposed by Medina [22]. Fig. 1 shows a flowchart of the simulated process.

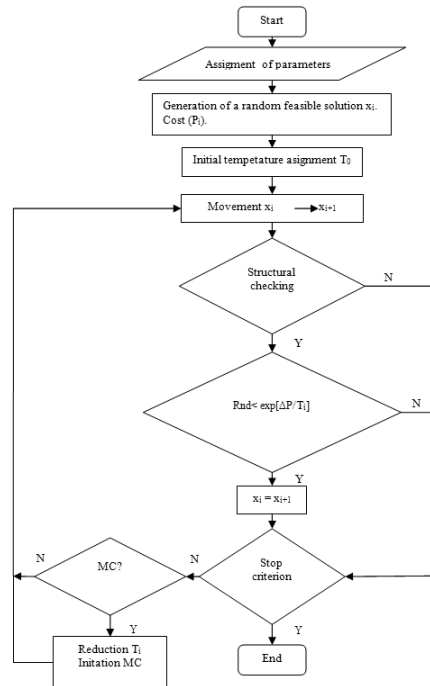


Figure 1: Flowchart of SA process.

## 2.2 Threshold accepting (TA)

The second algorithm used here is a local search method called “threshold accepting” (TA), which was initially described by Dueck and Scheuer [23]. This algorithm is a simplification of SA. TA accepts lower-quality solutions within a given threshold. In this case, the acceptance criterion is deterministic, in contrast to the case of SA. TA starts with a randomly generated solution with an initial threshold. The initial threshold decreases geometrically by the coefficient  $k$ , similar to the temperature of SA. The number of iterations or movements that are performed within the same threshold are called cycles. The algorithm ends when the value of the acceptance threshold reaches a small percentage of the initial values (typically 1%). The method requires the calibration of the initial threshold’s value, the cycle lengths and the threshold’s reduction coefficient  $k$ . Our adopted values are given in Section 4. As with SA, the initial value of the threshold was adjusted, as proposed by Medina [22].



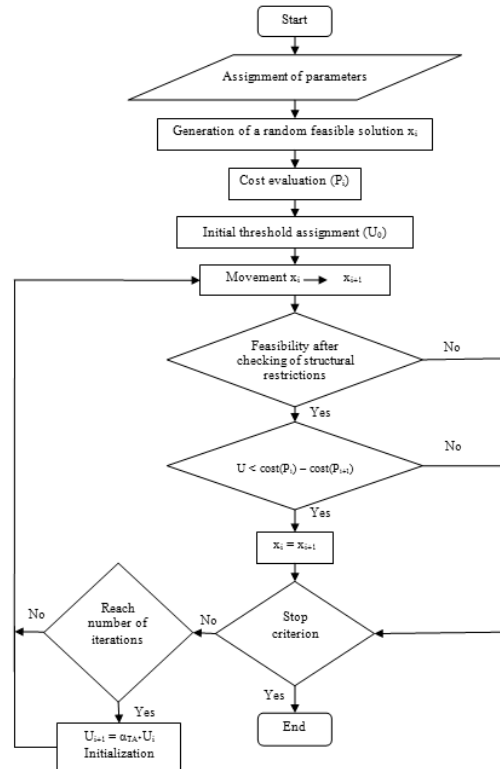


Figure 2: Flowchart of the TA process.

### 3 Results of the road bridge of prestressed concrete U-beams

This case study is a road bridge of prestressed precast beams with a double U-shape cross-section, which integrates a slab of reinforced concrete poured in-situ. It is a deck of 30–30 m of longitudinal spans, 12.00 m width and a beam spacing of 6 m (Fig. 3). The analysis includes 59 design variables. Figure 4 shows the main geometric variables considered in this analysis. The seven geometric variables include: the depth of the beam ( $h_1$ ), the thickness of the slab ( $e_4$ ), the width of the soffit of the beam ( $b_1$ ) and its thickness ( $e_1$ ), the width and the thickness of the flanges ( $b_3$  and  $e_3$ ) and the thickness of the webs ( $e_2$ ). Regarding the material strength, there are two variables defining the concrete grades used for the slab and the girder. There are 46 variables defining the standard reinforcement set-up in the beams and slab. The prestressing steel, which is formed by 0.6-inch strands, is defined by four variables: the number of strands in the top flanges, the number of strands in the first, second and third layer of the bottom flange. All variables of this analysis are discrete and can adopt a range of values, giving rise to  $1.6 \times 10^{65}$  possible solutions. This large number of solutions justifies the application of a



heuristic algorithm to find cost-effective results within a reasonable computational time.

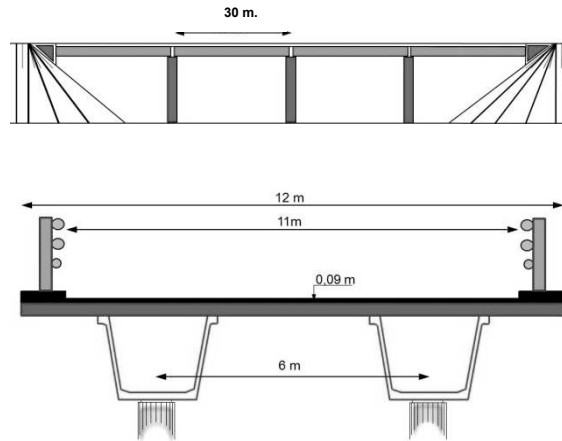


Figure 3: Longitudinal profile and cross-sectional geometry of a U-beam deck.

The parameters, as fixed values, do not affect the optimization process. The main parameters are divided into: geometric, loading, cost, reinforcement and exposure. These include the width of the deck, inclination of the webs, span length, slenderness of the beam, dead loads, transport distance, and steel types. The beam parameters are chosen to facilitate the adjustment of their design to the existing precast moulds. The durability requirements are those demanded by the concrete code EHE-08 [24]. Details of the parameters can be found in Martí *et al.* [17] (Table 1).

The structural constraints considered followed standard provisions for Spanish design of this type of structure [24, 25]. These include the verification of ultimate limit states of flexure, shear and torsion for the stress envelopes resulting from the dead loads, the traffic loads and the prestressing loads. The traffic load considered is a uniform distributed load of  $4 \text{ kN/m}^2$  and a point load of  $600 \text{ kN}$ . The stresses and strains were obtained using two models: a model for beam calculation in all the possible conditions before interacting jointly with the slab, including 20 bars and 21 sections; and another model for deck calculation including a grid with 103 bars and 84 sections. Thus, the structure is divided into elements that are only connected at their nodes. Deflections were limited to  $1/250$  of the free span length for the quasi-permanent combination. Fatigue of concrete and steel was considered.

SA was applied to a prestressed concrete precast double U-shape cross-section beam of 30-30 m longitudinal spans, 12.00 m width and a beam spacing of 6 m (Fig. 3). Partial safety coefficients of 1.50 for variable loading, 1.35 for permanent loading and 1.00 for prestress loading are used. Regarding materials, partial safety coefficients of 1.50 for concrete and 1.15–1.00 for passive-prestressing reinforcement are used.



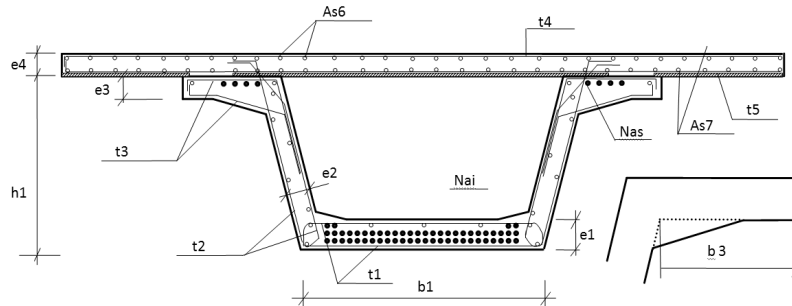


Figure 4: Arrangement of the geometric variables and the reinforcement.

Table 1: Inputs parameters for the analysis.

|                                      |                              |
|--------------------------------------|------------------------------|
| Loading parameters                   |                              |
| Concrete bridge barrier width        | 2 x 0.50 m                   |
| Thickness of the wearing surface     | tws = 0.09 m                 |
| Concrete bridge barrier loads        | 2 x 5.0 kN/m                 |
| Geometric parameters                 |                              |
| PC precast bridge width              | W = 12.00 m                  |
| Spacing between beams                | Sv = 6.00 m                  |
| Web inclination                      | 80°                          |
| Minimum beam slenderness             | L/18                         |
| Bearing center to beam face distance | 0.47 m                       |
| Reinforcement parameters             |                              |
| Passive reinforcing steel (B-500-S)  | fyk = 500 N/mm <sup>2</sup>  |
| Active prestressing steel (Y1860-S7) | fpk = 1700 N/mm <sup>2</sup> |
| Strand diameter                      | Φs = 0.6"                    |
| Beam surface reinforcement           | Φr = 8 mm                    |
| Strand sheaths                       | Levels 2 and 3               |
| Vertical slenderness of stirrups     | 200 (length/diameter)        |
| Cost parameters                      |                              |
| Transport distance (one way)         | Td = 50 km                   |
| Active prestressing steel crops      | 25%                          |
| Legislative and exposure parameters  |                              |
| Code regulation                      | EHE/IAP-98                   |
| External ambient conditions          | I Ib (EHE)                   |

The algorithm was programmed in Fortran 95 with a Compaq Visual-Fortran compiler. The initial temperature  $T_0$  was adjusted according to the method proposed by Medina [22]. The other heuristic parameters are the length of the Markov Chain of 2500, a reducing coefficient of 0.95, and a stopping criterion of nine runs and two chains without improvement.



Initially, the applied heuristic accepts solutions easily, increasing its difficulty as the process advances. The slope of the curve turns horizontal around the second 13000 (see Fig. 5). From this moment, 42 solutions are accepted in 5217 s; the equivalent to a solution every 124 s, which contrasts with a solution every 0.23 s in the earliest 4000 accepted solutions. The first part corresponds to the diversification phase, and the second to intensification.

The minimum result has a cost of 91412 €, including transport and beam placement. The depth of the beam is 1.64 m, the thickness of the slab is 0.22 m, the width of the soffit of the beam is 2.00 m, the thickness of the web is 0.10 m, the number of strands of 0.6" in diameter of the bottom flange is 50, and 1 in each top flange, the concrete in the slab is HA-40 and HP-45 in the beam. The prestressing and passive reinforcement set-up is shown in Fig. 4.

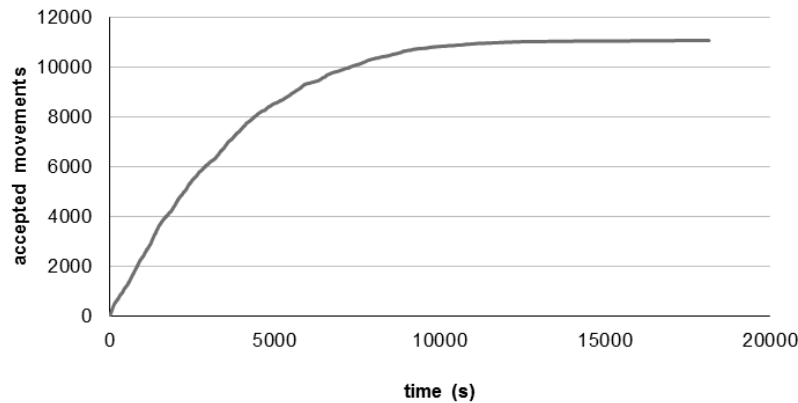


Figure 5: Number of movements accepted with respect to the computing time.

The amount of steel per square meter is: 11.9 kg for the active reinforcement, 52.4 kg for the passive reinforcement, 0.132 m<sup>3</sup> for the concrete in the beam and 0.22 m<sup>3</sup> for the concrete in the slab. The cost of the best solution is 245.73 € per square meter of the deck. The SA method indicates savings of about 5% with respect to a traditional design.

#### 4 Results of the slab bridge of prestressed concrete

The second study is a post-tensioned cast-in-place concrete slab deck with a concrete lightweight gull wing section based on Alcalá [26]. This type of bridge is typically used in road overpasses. The prestressing reduces the excessive longitudinal deformation occurring under constant loads, and to avoid cracking imposed by repetitive loads. The optimization case has spans of 20–36–20 m (see Fig. 6) and a total width of 11 m.

Regarding the deck slab, five variables describe (Fig. 7) the cross section, one defines the concrete type, and another specifies the amount of pretension. Furthermore, 26 variables define the passive reinforcement according to a logical





reinforcement scheme typical for this kind of deck, which is determined by the diameter and spacing of the bars. All the variables in this analysis are discrete. The lightening of the sections is arranged by circular voids, leaving the minimum constructive separations to the section faces, and a minimum web thickness.

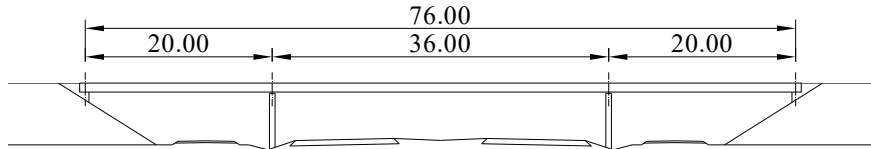


Figure 6: Longitudinal profile of the bridge.

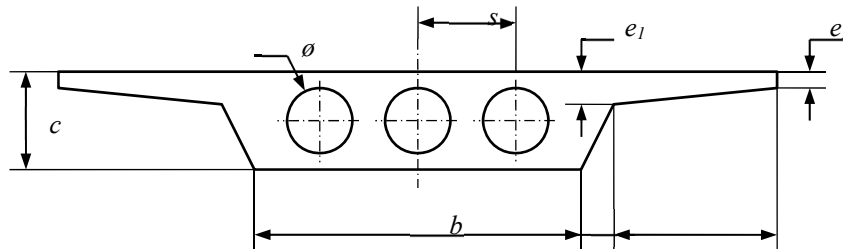


Figure 7: Cross section of the lightened deck with gullwing flanges.

The yield stress for prestressing and reinforcement is 1860 MPa and 500 MPa, respectively. The total number of prestressed cables can vary and its section is of 0.6 inches. Once the number of cables is set, they are distributed according to the number of webs. In addition, the anchors of the tendons should be spaced to fit the anchor cones at the end of the bridge. The tendons are arranged along a curved layout. The layout tries to achieve the maximum possible eccentricity in the critical sections of the deck.

The structural restrictions imposed on the deck are all mandatory for this type of structure. According to the Spanish code EHE-08 [25], these include the validation of the ultimate limit states of bending, shear, torsion, local bending in the flanges and fatigue. Regarding the serviceability limit states, the deflection, stresses and cracking are checked. The evaluation is conducted by a beam model consisting of 10 one-dimensional finite elements in each span. Within this model, defined loads according to the IAP [24] are used. In the implementation of the optimization algorithm, a subroutine checks the deck solution when it is entirely defined.

The proposed algorithm TA was programmed in Fortran 95 with a Compaq Visual-Fortran Professional 6.6.0 compiler. Each TA process needed around 2.8 hours in a Core 2 Quad CPU Q6600 processor (2.4 GHz, 3.21 GB RAM). In this case, the calibration of TA recommended a cycle length of 20000 iterations and a threshold reduction coefficient of 0.90. The stopping criterion was a threshold less



than 2% of the initial threshold, and no improvements during two consecutive cycles. 30 restarts were executed to obtain the best solution. Table 2 summarizes the characteristics of the best deck.

Table 2: Characteristics of the deck optimization.

|  | Result values |
|--|---------------|
| Span lengths (m)   | 20-36-20      |
| Deck depth (m)   | 1.45          |
| Slenderness  | 1/24.83       |
| Width of the soffit of the deck (m)                        | 4.40          |
| Flange thickness (m)                                       | 0.35-0.15     |
| Type of concrete   | 35 MPa        |
| Active reinforcement                                       | 8x16/0.6"     |
| Cost (€/m <sup>2</sup> deck)                               | 196.66        |
| Concrete (m <sup>3</sup> /m <sup>2</sup> deck)             | 0.58          |
| Active quantity (kg/m <sup>2</sup> deck)                   | 12.82         |
| Quantity of reinforcement (kg/m <sup>2</sup> deck)         | 63.52         |
| Quantity of reinforcement (kg/m <sup>3</sup> concrete)     | 109.79        |
| Longitudinal passive quantity (kg/m <sup>3</sup> concrete) | 19.06 (17%)   |
| Transverse passive quantity (kg/m <sup>3</sup> concrete)   | 90.73 (83%)   |

The optimized deck shows, compared to typical values of this bridge design, a reduced concrete volume, a reduced slenderness, a smaller amount of active steel, which is related to the generosity deck depth, and a higher amount of passive steel, especially in the transverse set-up. The TA method allows cost reductions of 7.5% with respect to a structural design based on the experience of the bridge designers.

## 5 Conclusions

This article has described the application of metaheuristic techniques to the automatic design of concrete structures. We presented two basic optimization algorithms: SA and TA. In particular, we showed the cost optimization of two prestressed concrete bridges that are very commonly used in public engineering works. The applied metaheuristic methods indicate that savings between 5–7% are achieved compared to traditional design methods. The results display the possible applicability of heuristic algorithms to the advanced automatic design of road bridges. It is important to note that the present model eliminates the need for experience-based guidelines of design. According to our experience, as part of this research, it is necessary to verify the functionality of the obtained structural designs in detail. Furthermore, we point out that the development of CAD software for inexperienced engineers can produce structurally questionable design solutions. Moreover, software development requires experienced engineers, who are able to identify functional disadvantages of the designed structures. The



generated solutions obtained directly from the computer program must not be taken as correct without questioning their feasibility.

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