

DESIGN OPTIMIZATION OF PRECAST-PRESTRESSED CONCRETE ROAD BRIDGES WITH STEEL FIBER-REINFORCEMENT BY A HYBRID EVOLUTIONARY ALGORITHM

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ABSTRACT

In this paper, the influence of steel fiber-reinforcement when designing precast-prestressed concrete (PPC) road bridges with a double U-shape cross-section is studied through heuristic optimization. A hybrid evolutionary algorithm (EA) combining a genetic algorithm (GA) with variable-depth neighborhood search (VDNS) is formulated to minimize the economic cost and CO₂ emissions, while imposing constraints on all the relevant limit states. The case study proposed is a 30-m span-length with a deck width of 12 m. The problem involved 41 discrete design variables. The algorithm requires the initial calibration. Moreover, the heuristic is run nine times so as to obtain statistical information about the minimum, average and deviation of the results. The evolution of the objective function during the optimization procedure is highlighted. Findings show that heuristic optimization is a forthcoming option for the design of real-life prestressed structures. This paper provides useful knowledge that could offer a better understanding of the steel fiber-reinforcement in U-beam road bridges.

Keywords: hybrid evolutionary algorithm, precast-prestressed concrete, steel fiber-reinforcement, U-shape cross-section.

1 INTRODUCTION

In this day and age, climate change and the gradual deterioration of our planet are both cause for concern. Greenhouse gases are one of the main reasons of climate change as they amplify changes in climate system. Flower and Sanjayan [1] have reported that the cement industry accounts for 5% of the total CO₂ emissions in the world. For concrete structures, savings in CO₂ emissions can be achieved not only by recycling [2] and by the use of novel materials, such as low-carbon cements and clinker substitutes [3, 4], but also by decreasing the unit CO₂ emissions of each structural material in the design stage and construction processes [5].

In this context, precast-prestressed concrete (PPC), pretensioned concrete beams with cast-in-situ slabs has been one of the most common forms of structural systems when building road bridges, given their cost effectiveness, especially when high production volumes are possible [6]. Standard PPC bridge beams are considered one of the key solutions to bridging problems in the short-to-medium-span range, typically ranging from 10 m to over 40 m. On the other hand, the stationary precasting industry offers optimal possibilities for steel fiber-reinforced concrete (SFRC) as a cement-based composite material. Although the use of SFRC allows for savings on assembling operations related to conventional reinforcement and for reductions in labor force, equipment use, and associated risks [7], steel fibers are often considered expensive. Additionally, reducing material weight through prestressing is essential due to elevation and transportation requirements. This is where structural optimization of this type of large and repetitive structures is an area of much research interest given the large amount of materials required in the manufacturing process.

PC beams optimization is a classical problem considered many years ago [8]; however, as Hernandez *et al.* [9] have suggested, most approaches for PC bridges found in the literature are not suitable for implementation in real-life engineering. While there is little research on optimization of PC structures [10–14], the literature includes numerous studies on optimizing real-life reinforced concrete [15–19]. Sarma and Adeli [20] reviewed research on cost optimization of concrete structures while Hassanain and Loov [21] did the same for concrete bridge structures. However, little attention has been paid to the CO₂ emissions optimization of PC structures [22, 23]. Regarding SFRC structures, optimization techniques have been employed in recent years in the design of fiber-reinforced concrete mixes [24, 25]. However, the literature includes very few works on the cost optimization of SFRC structures [26–28].

In this study, the interest of the authors in the CO₂ emissions and cost optimization of PPC road bridges focuses on the influence of steel-fiber reinforcement (SFR) on the optimal design of this type of structures. The PPC bridge system studied consists of two simply-supported U-beams with a cast-in-situ reinforced concrete slab for road traffic. The methodology followed in this study consisted in developing a computer evaluation module from the cross-section dimensions, materials and steel reinforcement. This module computed the CO₂ emissions of a solution and checked all the relevant limit states. The proposed optimal design method employs a hybrid evolutionary algorithm (EA) combining a genetic algorithm (GA) with variable-depth neighborhood search (VDNS) algorithm as an optimization tool. The objective function CO₂ emission is evaluated along with the economic cost during the bridge materials production and manufacture, transport and construction.

2 BRIDGE OPTIMIZATION

This paper studies the structural optimization of the CO₂ emissions of PPC road bridges with a double U-shape cross-section when using steel fiber-reinforcement concrete (SFRC). The optimization problem reduces the objective function f_1 eqn (1) while calculating f_2 eqn (2) and satisfying the restrictions eqn (3).

$$\text{CO}_2 = f_1(x_1, x_2, \dots, x_n) \quad (1)$$

$$C = f_2(x_1, x_2, \dots, x_n) \quad (2)$$

$$g_j(x_1, x_2, \dots, x_n) \leq 0 \quad (3)$$

The problem includes 41 discrete variables x_1, x_2, \dots, x_n chosen to describe the bridge design. These include the eight geometric variables (Fig. 1): the depth of the beam (h_1), the width of the beam soffit (b_1), the thickness of the bottom flange (e_1), the width of the top flanges of the beam (b_3), the thickness of the top flanges of the beam (e_3), the thickness of the webs (e_2), the thickness of the slab (e_4), and the spacing between beams (s_v). Two variables define the compressive strength of the concrete slab and beam. Besides, the design residual flexural strength of the concrete ($f_{R,3d}$) is needed to assess the normal stresses of the ULs. Prestressing is formed by four variables, which define the number the strands in the top flanges, in the bottom flange, in the second layer of the bottom flange, and in the third layer of the bottom flange. Finally, 26 variables are necessary to set the diameters, spacing, and lengths of the reinforcing bars which follow a standard set-up. In addition to the variables, 21 parameters are summarized in Table 1.

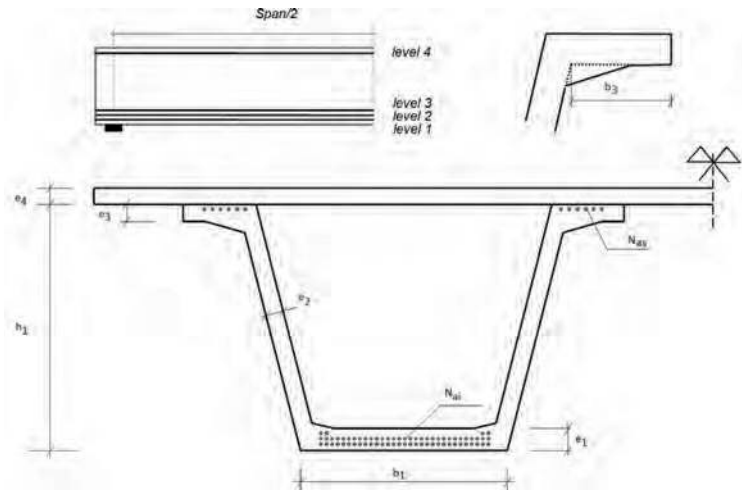


Figure 1: Main beam and slab variables.

Table 1: Parameters of the problem.

Parameters	Values
Geometric	
Bridge width	12.00 m
Web inclination	80°
Inclination of top flange tablet (1: ns3)	3
Top flange division	3
Inclination of the bottom flange tablet (1: ni3)	3
Bottom flange division	4
Bearing center to beam face distance	0.47 m
Minimum beam slenderness	L/18
Loads	
Concrete bridge barrier width	2 × 0.5 m.
Thickness of wearing surface	9 cm
Concrete bridge barrier loads	2 × 5.0 kN/m
Cost	
Transport distance	25 km
Active prestressing steel crops	25%
Reinforcement	
Passive reinforcing steel (B-500-S)	500 N/mm ²
Active prestressing steel (Y1860-S7)	1700 N/mm ²
Strand diameter	0.6"
Strand sheaths	Level 2 y 3
Vertical slenderness of the stirrups	200 (longitude/Φ)
Exposure scenario	
Exposure class	Iib (EHE)

Table 2: Unit prices and CO₂ emissions of the PC precast bridge.

Construction units	CO ₂ (kg)	Cost (€)
kg of reinforcing steel in the beams (B-500-S)	3.03	2.81
kg of reinforcing steel in the slab (B-500-S)	3.04	1.50
kg of active steel (Y-1860-S7)	3.13	3.62
kg of beam steel fiber	2.35	1.10
m of beam formwork	15.49	80.37
m ² of slab formwork	41.91	32.00
m ³ of slab concrete HA-25	282.08	70.00
m ³ of slab concrete HA-30	303.47	75.00
m ³ of slab concrete HA-35	303.47	80.00
m ³ of slab concrete HA-40	303.47	85.00
m ³ of beam concrete HP-35	332.21	130.81
m ³ of beam concrete HP-40	332.21	142.74
m ³ of beam concrete HP-45	332.21	152.10
m ³ of beam concrete HP-50	332.21	163.59

The objective function measures the CO₂ emissions generated during all the construction process, from the raw material acquisition to the completion of the work. The unit emission values considered were obtained from BEDEC database [29]. The environmental assessment can be obtained by multiplying the unit emission values (e_i) by the corresponding measurements (m_i) for the total of the construction units (r) eqn (4). Thus, several PPC bridges of SFR can be compared from the environmental point of view.

$$CO_2 = \sum_{i=1,r} e_i \times m_i \quad (4)$$

Likewise, the cost is evaluated for each unit price (p_i) according to the eqn (5). The total cost includes the material cost (concrete, prestressing steel, reinforcement and steel fibers) and the other construction units required to assess the bridge construction project (Table 2). Unit prices were obtained from a survey of Spanish contractors. The cost function is evaluated along with the CO₂ emissions to know the cost of the best environmental solution.

$$C = \sum_{i=1,r} p_i \times m_i \quad (5)$$

3 STRUCTURAL EVALUATION

This module calculates the stress envelopes and checks all the structural constraints of eqn (4) from a given structure. The constraints follow the standard provisions for the Spanish design of this type of structure [30, 31]. The design live load consists of three axis of 200 kN each (1.5 m distance between axes), superimposed with a uniform load of 4.0 kN/m². The dead load is a wearing surface of 0.09 m as well as a uniformly distributed load of 2×0.5 kN/m

for concrete bridge barrier rails installed along the edge of the deck. The combinations of actions include all construction stages. Laminated neoprene bearing pads are used to support the precast concrete bridge beams. A single support point is used at each end of the beam. The pads are designed to carry vertical loads and to accommodate horizontal movements of the bridge girders. They are also designed to deflect horizontally under shearing-type forces. The constraints check the serviceability and ultimate limit states that corresponds to this structural case. In addition, the geometric and constructability restrictions should be verified. This study accepts unfeasible solutions. These solutions are penalized, increasing the emission value. The loads and other structural analysis details are specified in Martí *et al.* [13]. Regarding the ULSs for flexure and shear, this paper applies the recommendations provided by the Annex 14 of the EHE-08 [31]. In addition, fatigue of concrete and steel was considered. Temporary deflections were limited to 1/250 of the free span length for the frequent combination. Further, time-dependent deflections were limited to 1/1000 of the free span length for the quasi-permanent combination. Finally, the durability limit state requires compliance of the service working life.

4 HEURISTIC ALGORITHM

The EA combines the parallel search inherent in the GA with the local search. This hybridization was called memetic algorithm [32]. The idea of improving the population by performing a local search is based on the diversification and intensification strategies [33]. Krasnogor and Smith [34] provided examples of the use of this algorithm.

This paper proposes a variant of the Very Large-Scale Neighborhood Search (VLSN) algorithm for the local search strategy. This variant belongs to a class of heuristic known as VDNS [35]. VDNS carries out a local search that random change one variable until a local optimum is reached. The key idea is to adaptively change the size of the neighborhood so that it can effectively traverse larger search space within a reasonable amount of time. The new solution is chosen if it improves the previous one. Then, the movement is enlarged to escape the local optimum. This process is repeated for a predefined number of movements. This study uses the hybrid MA-VDNS, increasing the movement in one variable when a number of movements without improvement are reached.

MA-VDNS algorithm starts with the random generation of a population of 500 solutions. Each solution is improved by a VDNS local search. The algorithm begins with one variable. The number of variables is increased after 10 consecutive movements without improvement. A maximum of eight variables can be simultaneously changed, which corresponds to 20% of the variables. Then, a GA is applied to this new improved population. A probability of 0.50 and elitist selection is used as the genetic parameters. Penalties are not applied during the VDNS local search to avoid the early divergence of the algorithm. A VDNS local search is carried out to each generation. The algorithm continues for a maximum of 150 generations.

5 RESULTS

In this Section, we examine the results from numerical experiments involving MA-VDNS optimization applied to a PC precast road bridge. The algorithm and the Structural evaluation module were coded in *Intel® Visual Fortran Compiler Integration for Microsoft Visual Studio 2008*. A personal computer with an INTEL® Core™ i7 CPU X980 processor with 3.33 GHz needed about 1500 min to run the proposed MA-VDNS algorithm. The algorithm was run nine times and discarding the extreme upper and lower values, the minimum, mean and deviation values for the emissions, the cost, the main geometry of the solutions and the active

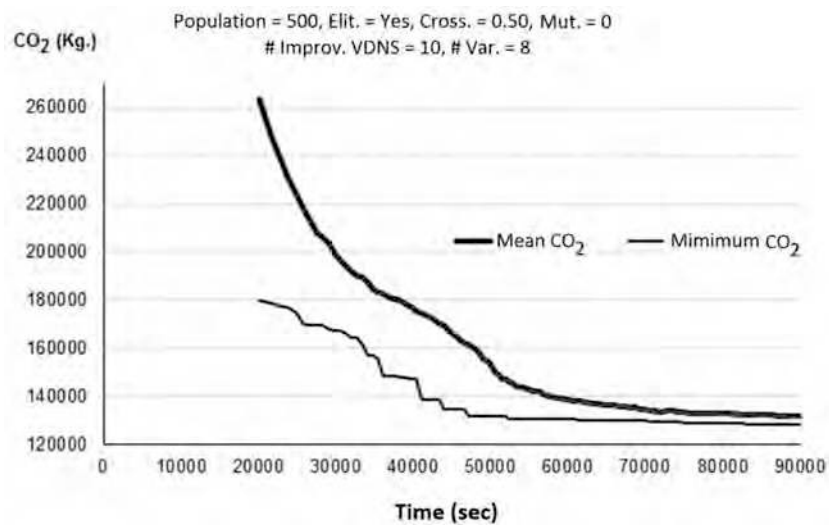


Figure 2: Typical convergence of the mean and minimum CO₂ emissions curves with the calculation time.

steel were obtained. Table 3 summarizes mean solutions for emissions, comparing them with the average values obtained by optimizing the cost. Note here that the average value of the variables does not necessarily take the allowed standard values of each variable.

Figure 2 shows a typical evolution of the CO₂ emissions with the computing time for the algorithm. Note the difficulty in obtaining the first 500 solutions, which takes about 20,000 seconds to show results. This is due to the complexity of the algorithm to achieve a combination of variables that fit the prestress in order to find a feasible solution – around 40 seconds-. VDNS algorithm determines a local optimum that is the starting point for the GA. Similar behavior were observed on all the processes, where the average values were declining rapidly at first, then were slower, and finally remained fairly constant. It is noted that after 52,000 seconds – generation number 50 –, the best CO₂ emission solution have improved little (1.7%), and this improvement is practically imperceptible after 75,000 seconds – generation number 106 –, up to the end (0.5%).

This hybrid algorithm allows two types of improvements, the genetic one for the population of 500 solutions and the local one for each in the 150 genetic combinations. Genetic improvements occur over generations, with an average of 33 genetic improvements. VDNS algorithm gives an average of 33 improvements that are originated from the first generations. Figure 3 shows their differences in performance, showing the last generation where there has been a local and a genetic improvement.

From Table 4, which summarizes the mean geometric characteristics of the solutions, the following can be noted:

The average depth of the beam has a ratio $L/19.2$, always lower than $L/18$. The average thickness of the slab is very small, 0.17 m, aiming to reduce the dead weight of the deck, but necessary to resist bending forces of the variable loads considered.

The thickness of the webs and the top flange of beam are the minimum allowed to reduce the weight, but at the same time, to contain the active reinforcements in the top flange of beam as well as resist shear and torque forces in webs. The width of top flanges of the beam is short, averaging a value of 0.29 m, enough space to locate the active reinforcement. The top

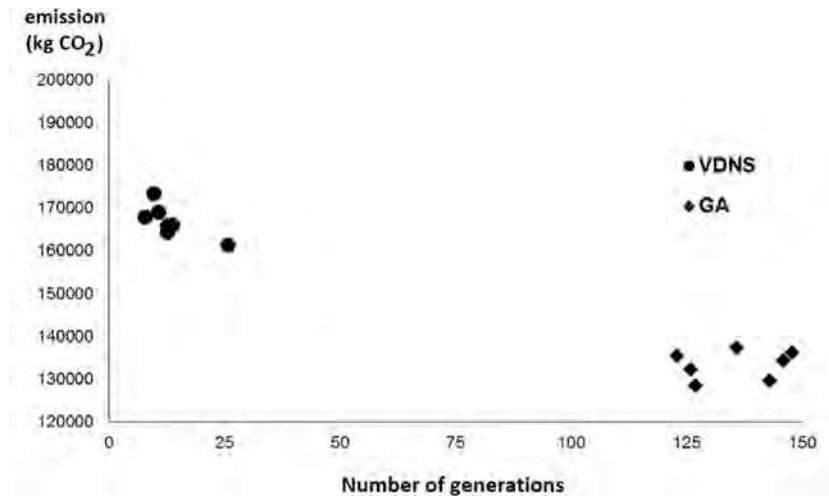


Figure 3: Last generation where there has been a VDNS/GA improvement.

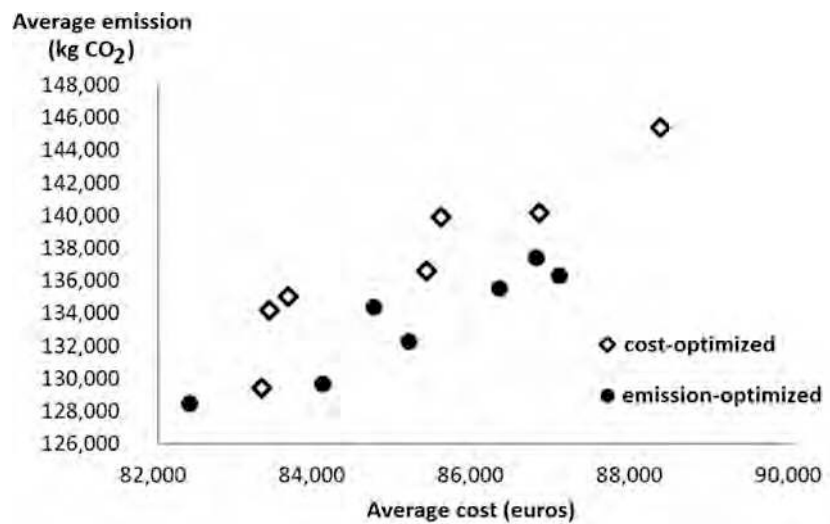


Figure 4: Relationship between CO₂ emissions and cost.

flange of the beam collaborates with the slab to increase the depth of both of them to have better resistance to the variable loads transverse bending; however, its magnitude is not very limited.

A high-quality concrete is not required. The average characteristic compressive strength of concrete in the beam f_{ck} is 37 MPa, with a maximum value of 50 MPa; this value in the deck is 29 MPa, with a maximum of 40 MPa. Steel fibers contribute to the strength of concrete f_{R3k} with 5.1 MPa, with a maximum value of 7.0 MPa. The average number of strands in the bottom flange is 47, distributed in two levels, and two strands in the top flange, which is the minimum number to avoid cracking when the bottom strands of the beam are prestressing in the precast plant. The average spacing between beams S_v is 5.56 m. This spacing is less than

Table 3: Measurements of average cost (1) and average emission (b) optimized solutions.

	CO ₂ (kg)	Cost (€)	<i>h</i> 1 (m)	<i>e</i> 4 (m)	<i>b</i> 1 (m)	<i>b</i> 3 (m)	<i>e</i> 1 (m)	<i>e</i> 2 (m)	<i>e</i> 3 (m)	<i>H</i> <i>v</i> (MPa)	<i>H</i> <i>l</i> (MPa)	<i>p</i> 1 (n)	<i>p</i> 2 (n)	<i>p</i> 3 (n)	<i>p</i> 4 (n)	<i>S</i> (m)	<i>f</i> <i>R</i> 3 <i>k</i> (MPa)
(1)	137,245	85,197	1.58	0.19	1.39	0.28	0.17	0.10	0.14	36	31	25	20	0	3	5.54	5.1
(2)	133,409	85,210	1.56	0.17	1.53	0.29	0.15	0.10	0.11	37	29	27	20	0	2	5.56	5.1

half the bridge width (6 m), to better fit the variable load, in which the point load is displaced from the end.

Figure 4 shows the average value variation in the minimum cost and CO₂ emissions for the objective functions. The results depicts the relationship between the values for the emissions and cost, which indicates that, as a rule of thumb, reducing cost by 1 Euro results in savings of 1.61 kg in CO₂ emissions (Table 3).

6 CONCLUSIONS

A hybrid EA combining a GA with VDNS is used to minimize the economic cost and CO₂ emissions, on PPC road bridges, typically formed by two isostatic beams, with a double U-shaped cross-section. This algorithm was run several times providing little variation from the mean of the values found, achieving good solutions, even in the case of a single run. The optimized bridge solutions with 30 m span length show that the average depth of the beam is 1/19.2 in relation with the span. The average slab thickness is only 0.17 m so that optimized solutions tend to minimize the weight of the structure because the web thickness and the slab thickness are also very low. The characteristic compressive strength of concrete in the beam is higher than in the slab, with average values of 37 MPa and 29 MPa, respectively, without it having been necessary to reach the maximum possible resistance in any of the cases. Similarly, steel fiber reinforcement contributes to the concrete tensile strength with a value of 5.1 MPa, but far away from the limit. Finally, the analysis reveals that CO₂ emissions and cost are closely related and, as a rule of thumb, a euro reduction in cost results in savings of 1.61 kg in CO₂ emissions.

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